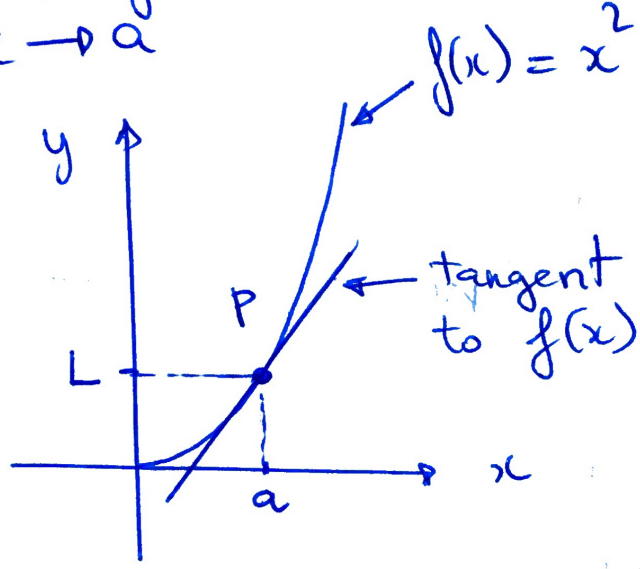
LIMITS.

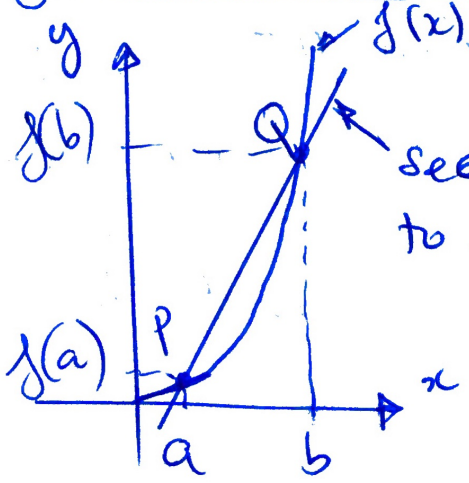
Sep., 13

Definition: $f(x)$ is defined around all x near $x=a$, if $f(x)$ is arbitrary close to some value L for all x close to $x=a$ (but not including $x=a$), then we say.

$$\lim_{x \to a} f(x) = L$$



Tangent line and secante line.



⇒ it crosses the curve $f(x)$ at 2 points (P, Q)

We know that the equation of a line is

$y = mx + k$, where m is the ~~spt~~ slope.

Here, the slope of our secante is:

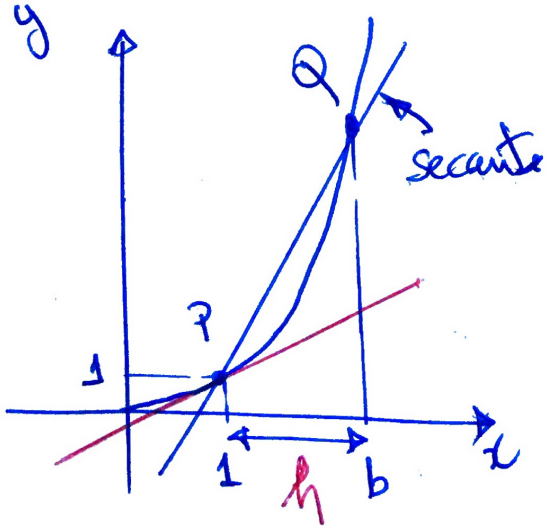
$$m = \frac{y_Q - y_P}{x_Q - x_P}$$

So, using the set of 2 points P and Q, we can find the equation of the secante line.

* What if ~~we~~ we are asked to give the equation of the tangent ~~ligne~~ passing ~~by~~ through the point P (1,1)?

⇒ We know that $y = x^2$, and let's say that Q is distant to P by $x = h$.

It means that $Q = (1+h, (1+h)^2)$



$$\text{slope } m = \frac{(1+h)^2 - 1}{1+h-1}$$

$$= \frac{1+2h+h^2-1}{h}$$

$$m = 2+h$$

$b = 1+h$

h	0.1	0.01	0.001	0.0001
m	2.1	2.01	2.001	2.0001

$\lim_{h \to 0} 2+h = 2.$

Mathematically, we write:

$$\lim_{h \to 0} \frac{(1+h)^2 - 1}{1+h-1} = \lim_{h \to 0} \frac{(1+h)^2 - 1}{h} = 2.$$

Our slope m is then 2:

$\Rightarrow y = 2x + k$, then using $P(1,1)$, we can have the equation of the tangent line

$$\boxed{y = 2x - 1}$$

Average rate of change $\Rightarrow$ slope of sec.
Instantaneous rate of change $\Rightarrow$ slope of tangent.

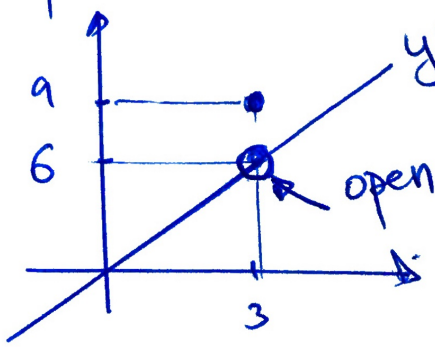
The limit of a function

It is written as: $\lim_{x \to a} f(x) = L$

It is red as: limit of $f(x)$ as x approaches a is L.

example 1: Consider the piece-wise function

$$f(x) = \begin{cases} 2x & x < 3 \\ 9 & x = 3 \\ 2x & x > 0 \end{cases}$$



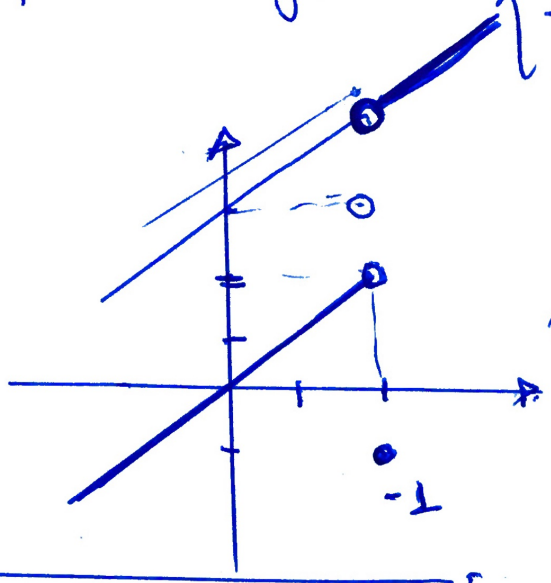
x	2.9	2.99	2.999	3	3.001	3.01
f(x)	5.8	5.98	5.998	0	6.002	6.02

$\rightarrow$ 6 $\leftarrow$

$\therefore f(3) \neq 6.$

The limit as x approaches 3 of $f(x)$ is 6.

Example 2. $f(x) = \begin{cases} x & x < 2 \\ -1 & x = 2 \\ x+3 & x > 2 \end{cases}$



x	1.9	1.99	2.001	2.01	2.1
$f(x)$	1.9	1.99	5.001	5.01	5.1

$\lim_{x \to 2} f(x) = DNE$