One-sided limits / two-sided limits

- Left-hand limit: $\boxed{\lim_{x \to a^-} f(x) = K}$

When the value of $f(x)$ gets closer to K when $x < a$ and x moves closer to a.

- Right-hand limit $\boxed{\lim_{x \to a^+} f(x) = L}$

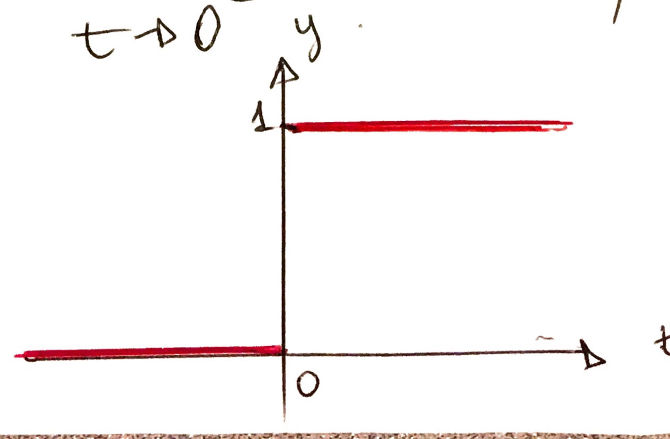
When the value of $f(x)$ gets closer to L when $x > a$ and x moves closer to a..

Theorem: $\lim_{x \to a} f(x) = L$ if and only if.

$$\lim_{x \to a^-} f(x) = L \text{ and } \lim_{x \to a^+} f(x) = L$$

Example 1: Estimate the value of the following limits.

$$\lim_{t \to 0^+} H(t) \text{ and } \lim_{t \to 0^-} H(t) \text{ where } H(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$$

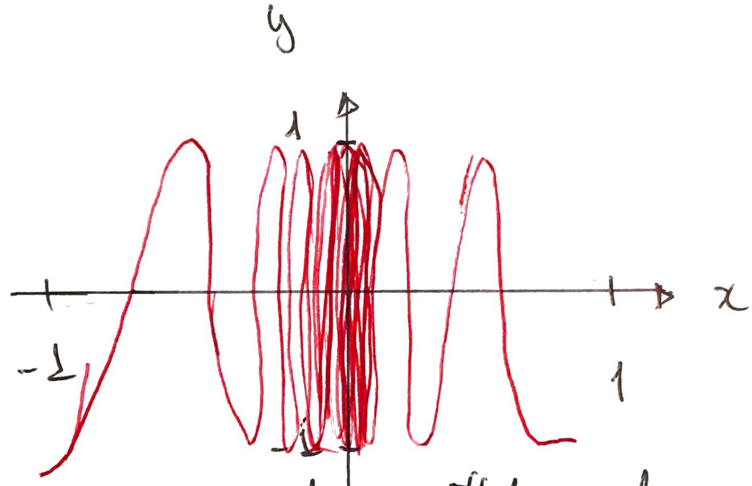


Solution: $\lim_{t \to 0^+} H(t) = 1 \qquad \lim_{t \to 0^-} H(t) = 0.$

example 2 Estimate the value of the following limits:

$$\lim_{x \to 0^+} \cos\left(\frac{\pi}{x}\right) \qquad \lim_{x \to 0^-} \cos\left(\frac{\pi}{x}\right).$$

Solution:

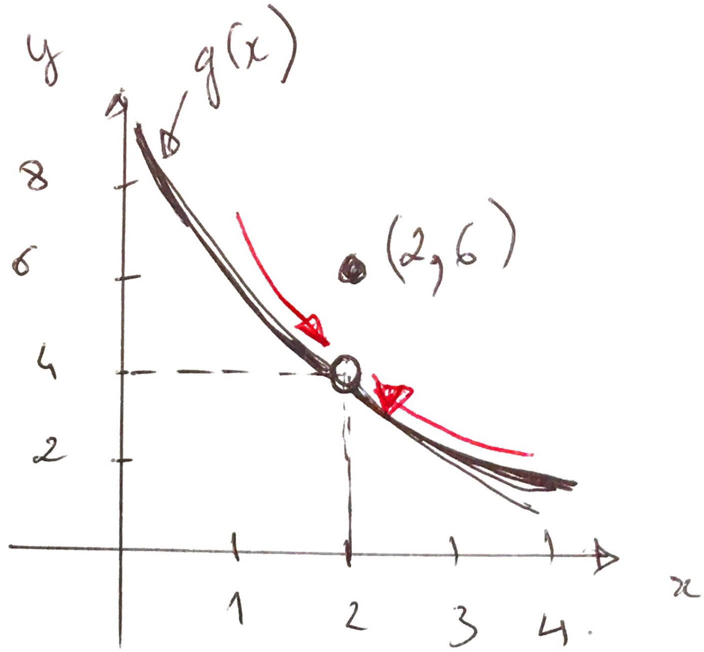


The function does not settle down to a single number on either side of $x=0 \Rightarrow$ DNE.

example 3 Estimate the value of the following limits

$$\lim_{x \to 2^+} g(x) \qquad \lim_{x \to 2^-} g(x) \qquad g(x) = \begin{cases} \dfrac{x^2+4x-12}{x^2-2x} & x \neq 2 \\ 6 & x = 2 \end{cases}$$

Solution:



$$\lim_{x \to 2^+} g(x) = 4 \qquad \lim_{x \to 2^-} g(x) = 4$$

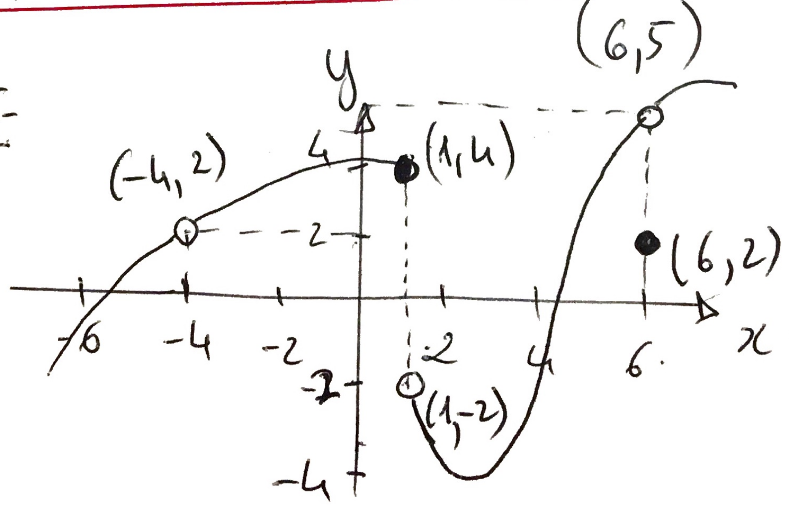
Given a function $f(x)$. if,

$$\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x) = L$$

then the normal limit will exist and

$$\lim_{x \to a} f(x) = L$$

example 4:



a) $f(-4)$? b) $\lim_{x \to -4^-} f(x)$? c) $\lim_{x \to -4^+} f(x)$? d) $\lim_{x \to -4} f(x)$.

a) DNE b) 2 c) 2 d) 2

e) $f(1)$? f) $\lim_{x \to 1^-} f(x)$? g) $\lim_{x \to 1^+} f(x)$? h) $\lim_{x \to 1} f(x)$.

e) 4 f) 4 g) −2 h) DNE.

$\boxed{\text{Limit laws}}$

$$\lim_{x \to a} f(x) = f(a).$$

* polynomial
* e^x
* $\sin(x)$, $\cos(x)$.

Let be $\lim_{x \to a} f(x) = L$ and $\lim_{x \to a} g(x) = M$

* $\lim_{x \to a} [f(x) + g(x)] = L + M$
* $\lim_{x \to a} [f(x) - g(x)] = L - M$.
* $\lim_{x \to a} [f(x) \cdot g(x)] = L \cdot M$
* $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{L}{M}$ except when $M = 0$
* $\lim_{x \to a} [f(x)]^n = L^n$ ← integer.
* $\lim_{x \to a} [f(x)]^{1/n} = L^{1/n} = \sqrt[n]{L}$ $f(x) \geq 0$ around a.

example: 5.

* $\lim\limits_{x \to 3} (x^2+2x+1) = \lim\limits_{x \to 3} x^2 + \lim\limits_{x \to 3} 2x + \lim\limits_{x \to 3} 1$

$= 9 + 2(3) + 1 = 16$

* $\lim\limits_{x \to 2} \sqrt[3]{4x^2+3x+5} = \lim\limits_{x \to 2} (4x^2+3x+5)^{1/3}$

$= \sqrt[3]{\lim\limits_{x \to 2} 4x^2 + \lim\limits_{x \to 2} 3x + \lim\limits_{x \to 2} 5} = \sqrt[3]{4(4)+3(2)+5}$

$= \sqrt[3]{27} = 3$

* $\lim\limits_{x \to 1} \frac{x^3-x^2}{x-1} = \frac{0}{0}$. DNE.

→ Simplify the function.

$\lim\limits_{x \to 1} \frac{x^3-x^2}{x-1} = \lim\limits_{x \to 1} \frac{x^2\cancel{(x-1)}}{\cancel{(x-1)}} = \lim\limits_{x \to 1} x^2 = 1$

Here, the domain of the function is

$D = \{x \in \mathbb{R} \mid x \neq 1\}$