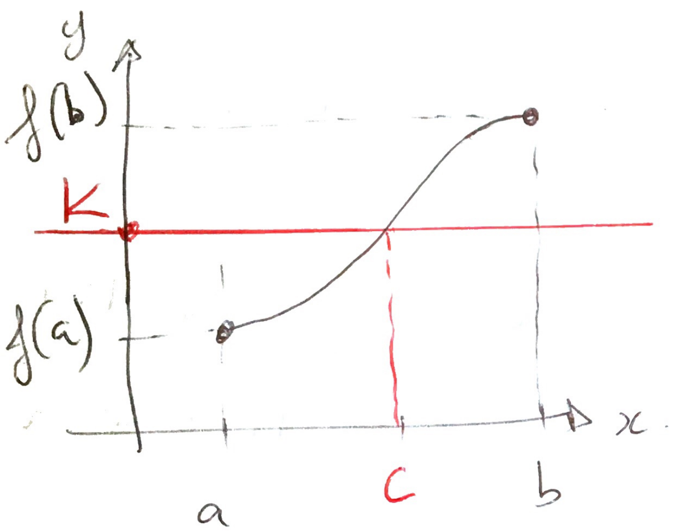
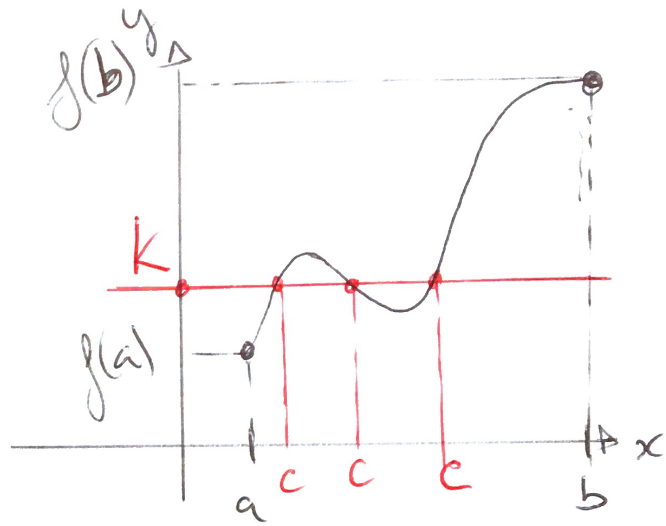
# Intermediate Value theorem   Sept. 19

Theorem: Let $a < b$ and let $f$ be a continuous function at all points $a \leq x \leq b$. For any $K$ between $f(a)$ and $f(b)$, there exists at least one number $a < c < b$ for which $f(c) = K$.



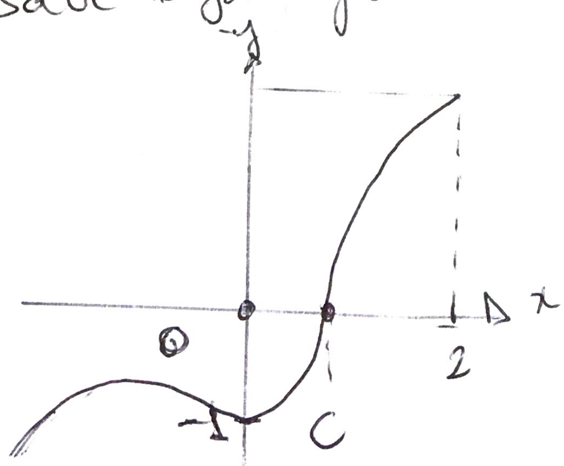




THERE MIGHT BE MORE THAN ONE C.

Example 1: $f(x) = 3x^3 + x^2 - 1$. Show that $f(x)$ has at least one root between $[0, 2]$

Polynomial $\Rightarrow$ continuous on $[0, 2]$.

$\Rightarrow$ Solve for $f(x) = 0$




$f(0) = -1$

$f(2) = 11$

$-1 \leq 0 \leq 11$

$f(a) \leq 0 \leq f(b)$.

$\Rightarrow$ IVT $\exists c, 0 < c < 2$. for which $f(c) = 0$

$f(1) = 1 \quad f\left(\frac{1}{2}\right) = -\frac{5}{8}$

$f\left(\frac{1}{2}\right) < 0 < f(1)$.

$\frac{1}{2} < c < 1$.

example 2: $f$ is a continuous function.

$f(-2) = 3$ and $f(1) = 6$.

Which of the following is guaranteed by the IVT?

~~between 3 an 6~~

a) $f(c) = 4$ for at least one $c$ between 3 and 6 ✗

b) $f(c) = 0$ for at least one $c$ between $-2$ and $1$ ✗

c) $f(c) = 0$ for at least one $c$ between 3 and 6 ✗

d) $f(c) = 4$ for at least one $c$ between $-2$ and $1$ ✓

example 3: Show that $f(x) = x - 1 + \sin\left(\frac{\pi x}{2}\right)$ has a zero in $0 \leq x \leq 1$. (Use of IVT).

- Testing endpoints of the interval:

$f(0) = 0 - 1 + \sin(0) = -1 < 0$.

$f(1) = 1 - 1 + \sin\frac{\pi}{2} = 1 > 0$

So, we know that somewhere between 0 and 1 the function $f(x)$ is equal to zero.

BUT, in order to apply IVT, we have to show that the function is continuous.

- $x-1$ is continuous. (polynomial).
- $\sin\left(\frac{\pi x}{2}\right)$ is continuous (trig.)

Hence $f(x)$ is continuous in $[0,1]$

- ~~Let a=0~~ Since the function is continuous, we know that there is a point $c \in [0,1]$ so that $f(c)=0$

~~Bisection method to find a zero of $f(x)$.~~

~~$f(x) = x-1+\sin\left(\frac{\pi x}{2}\right)$~~

Average rate of change on an interval $[a,b]$

The average rate of change of $f(x)$ on $[a,b]$ is $\dfrac{f(b)-f(a)}{b-a}$.

The Instantaneous rate of change at a point a.

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The instantaneous rate of change is given by

$$\lim_{b \to a} \frac{f(b) - f(a)}{b - a} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$