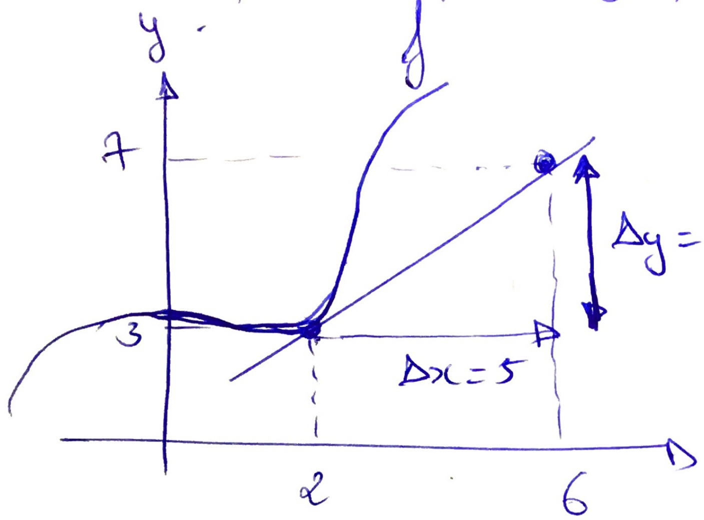
Derivatives

Sep 22.

The derivative of a function gives us the slope of the line tangent to the function at any point of the graph.



Find $f'(2)$

$\frac{\Delta y}{\Delta x} = \frac{3}{5}$ is the slope, and is the derivative $f'(2)$.

Derivative using the limit definition.

Let $a \in \mathbb{R}$ and let $f(x)$ be defined on an open interval that contains a.

- The derivative of $f(x)$ at $x = a$ is denoted $f'(a)$ and is defined by

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$ is it exists

- When that limit exists, the function $f(x)$ is said to be differentiable at $x = a$.

- We can equivalentely define the derivative $f'(a)$ by the limit.

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}.$$

example 1. find the derivative of $f(x) = \frac{1}{x}$. using limit definition.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}.$$

$$= \lim_{h \to 0} \frac{\frac{x - (x+h)}{x(x+h)}}{h} = \lim_{h \to 0} \frac{-h}{h\, x(x+h)} =$$

$$f'(x) = \lim_{h \to 0} \frac{1}{x(x+h)} = -\frac{1}{x^2}$$

- Differentiation using formula.

$f(x)$	$f'(x)$
U^n	$n U' U^{n-1}$
x^n	$n x^{n-1}$.
$\sin(x)$	$\cos x$

$f(x)$	$f'(x)$
$\cos(x)$	$-\sin(x)$
$\tan(x)$	$1 + \tan^2 x$.
e^x	e^x
$a^x (a>0)$	$a^x \ln(a)$

laws of derivative:

If f' and g' exists.

$(f \pm g)' = f' + g'$

$(cf)' = c f'$

$(f \cdot g)' = f' \cdot g + f g'$ product rule.

$\left(\frac{f}{g}\right)' = \frac{f'g - f g'}{g^2}$ quotient rule.

example 2: $f(x) = |x| x$. Find the derivative of $f(x)$ at $x = 0$

$f'(x) = x' |x| + x (|x|)'$.

Product rule does not apply because derivative of $|x|$ does not exist. Therefore, we take the limit definition.

$$f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0} \frac{h|h| - 0}{h}.$$

$$f'(0) = \lim_{h \to 0} |h| = 0$$

example 3 : Find the derivative of .

a) $y = x^{45} - x^{-45}$

$y' = (x^{45})' - (x^{-45})' = 45x^{44} + 45x^{-46}$.

b) $h(x) = e^{x} + \cos(x) - 2x\sqrt{x} \Rightarrow (e^{x})' + (\cos x)' - (2x\sqrt{x})'$

$h'(x) = e^{x} - \sin(x) - 3x^{1/2}$.

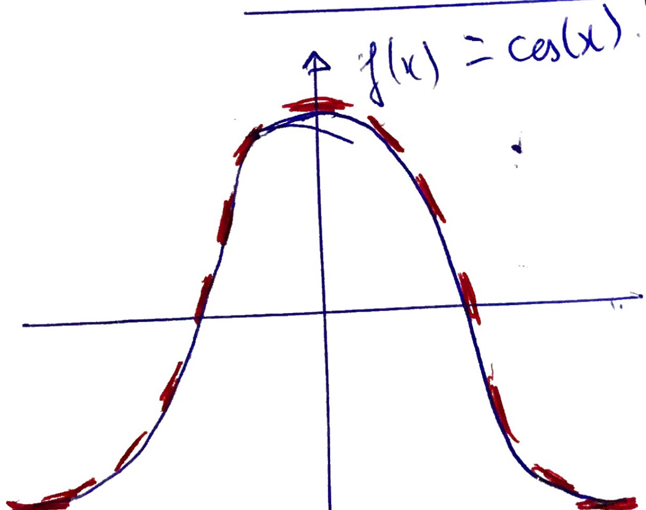
c) $f(x) = x \sin x$.

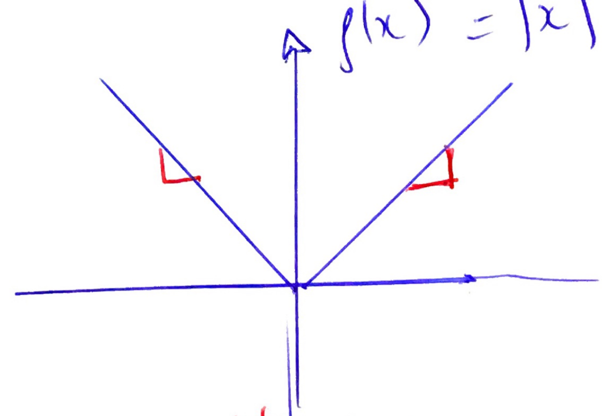
$u = x \quad u' = 1$

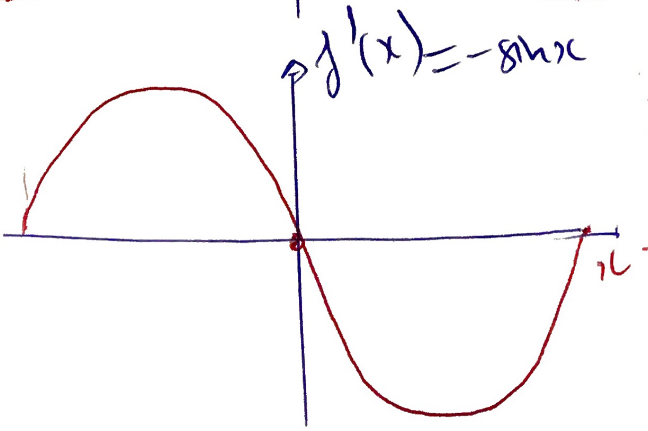
$v = \sin x \quad v' = \cos x$.

$f'(x) = x\cos x + \sin x$.

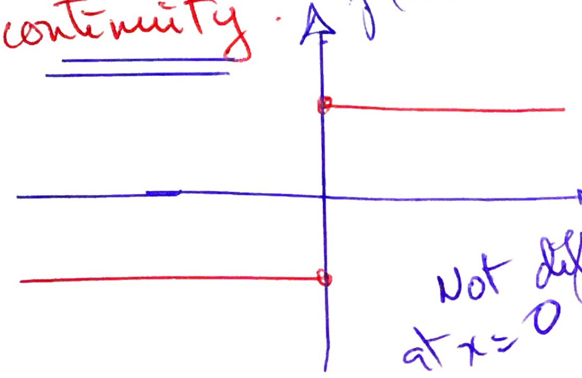
Sketch of the graph of f' given the graph of f .







Necessary condition for having derivative is continuity .



Not differentiable at $x = 0$