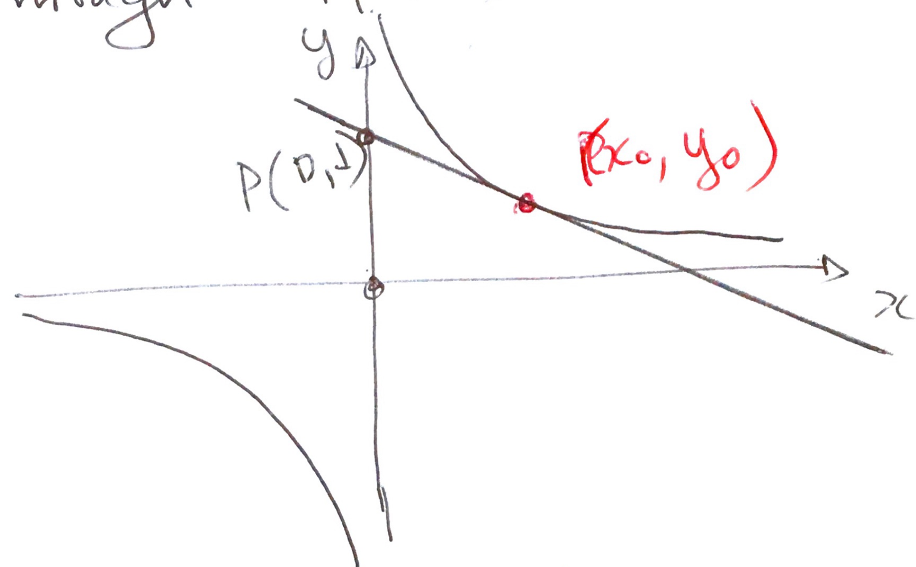
Tangent lines.

Find tangent lines to the curve $y = \frac{1}{x}$.
passing through $P(0,1)$



First, let's assume that the tangent is passing through the graph of $\frac{1}{x}$ at (x_0, y_0)

$\Rightarrow$ $y_0 = \frac{1}{x_0}$

$f'(x) = -\frac{1}{x^2}$ $\Rightarrow$ slope of the tangent ~~of~~ at any point of the graph. So at (x_0, y_0)

$f'(x_0) = -\frac{1}{x_0^2}$ ~~slop tangent~~ $\Rightarrow$ slope of the tangent at (x_0, y_0)

On the other hand, the definition of a slope is:

$m = \frac{y - y_0}{x - x_0}$ at ~~the~~ point (x_0, y_0).

$y - y_0 = -\frac{1}{x_0^2}(x - x_0)$ is the equation of the tangent line.

$y - \frac{1}{x_0} = -\frac{1}{x_0^2}(x - x_0)$

$$\boxed{y = -\frac{1}{x_0^2}x + \frac{2}{x_0}}$$

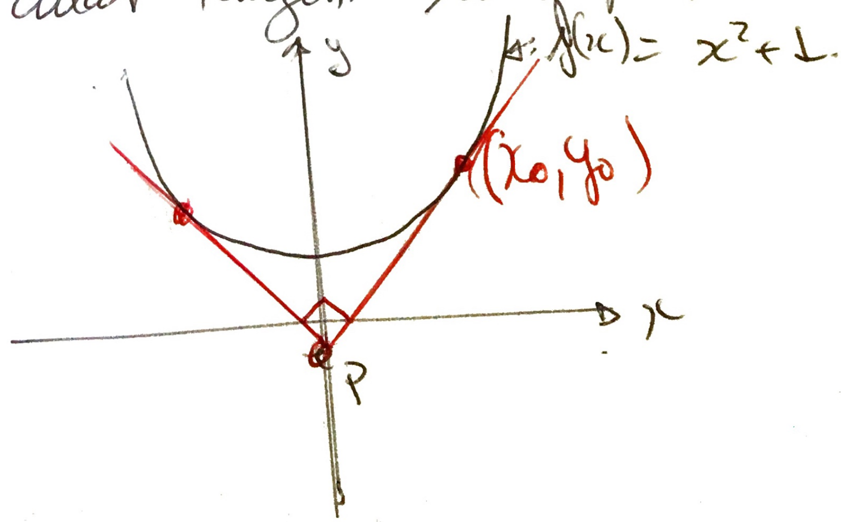
Now, plug the point $P(0,1)$ in that equation:

$1 = -\frac{1}{x_0^2}(0) + \frac{2}{x_0} \Rightarrow \boxed{x_0 = 2}$

$$\boxed{f(2) = \frac{1}{2} = y_0}$$

$f(x) = x^2 + 1$.

Find a point on y-axis from which we can draw two perpendicular tangent lines to the curve.



(x_0, y_0) is the tangent point on the curve.

$y_0 = x_0^2 + 1$

$f'(x) = 2x \Rightarrow$ slope of the tangent line at any point of $f(x)$.

$f'(x_0) = 2x_0 \Rightarrow$ slope of the tangent line at x_0

$y - y_0 = m(x - x_0)$

$y - (x_0^2 + 1) = 2x_0(x - x_0)$ Equation of tangent line.

$y = 2x_0 x - 2x_0^2 + x_0^2 + 1 = 2x_0 x - x_0^2 + 1$

$y_p = 2x_0 x_0 - x_0^2 + 1 \Rightarrow y_p = 1 - x_0^2$

$x_0^2 = 1 - y_p \rightarrow x_0 = \pm\sqrt{1 - y_p}$

$m_1 = 2\sqrt{1 - y_p}$

$m_2 = -2\sqrt{1 - y_p}$

$m_1 = -\frac{1}{m_2} \Rightarrow m_1 \cdot m_2 = -1.$

$\Rightarrow 2\left(\sqrt{1 - y_p}\right)\left(-2\sqrt{1 - y_p}\right) = -1.$

$-4(1 - y_p) = -1$

$1 - y_p = \frac{1}{4} \Rightarrow \boxed{y_p = \frac{3}{4}}$

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