Math 104 section 108 Homework week 5 Solutions

Daniel Rakotonirina

October 20, 2017

First Name: _____ Last Name: _____

Student Number: _____

Exercise 0.1. Differentiate the function $(\arcsin x)^{\arccos x}$. Assume $0 < x < 1$. (2 marks)

Solution: (0.5 mark for taking ln, 1 mark for taking derivative correctly, 0.5 mark for a complete solution)

We use logarithmic differentiation

$$\begin{aligned}
y &= (\arcsin x)^{\arccos x} \\
\ln(y) &= \ln[(\arcsin x)^{\arccos x}] = \arccos x \ln(\arcsin x) \\
\frac{d}{dx}(\ln y) &= \frac{d}{dx}[\arccos x \ln(\arcsin x)] \\
\frac{y'}{y} &= \frac{-1}{\sqrt{1-x^2}} \cdot \ln(\arcsin x) + \arccos x \cdot \frac{1}{\sqrt{1-x^2}} \cdot \frac{1}{\arcsin x} \\
y' &= y\left(\frac{-1}{\sqrt{1-x^2}} \cdot \ln(\arcsin x) + \arccos x \cdot \frac{1}{\sqrt{1-x^2}} \cdot \frac{1}{\arcsin x}\right) \\
y' &= (\arcsin x)^{\arccos x}\left(\frac{-1}{\sqrt{1-x^2}} \cdot \ln(\arcsin x) + \arccos x \cdot \frac{1}{\sqrt{1-x^2}} \cdot \frac{1}{\arcsin x}\right)
\end{aligned}$$

Exercise 0.2. Differentiate the function $y = (1-3x)^{\cos(x)}$. (2 marks)

Solution: (0.5 mark for taking ln, 1 mark for taking derivative correctly, 0.5 mark for a complete solution)

We still use logarithmic differentiation

$$\begin{aligned}
\ln y &= \ln\left[(1-3x)^{\cos(x)}\right] = \cos(x)\ln(1-3x) \\
\frac{d}{dx}(\ln y) &= -\sin(x)\ln(1-3x) + \cos(x)\frac{-3}{1-3x} \\
\frac{y'}{y} &= -\sin(x)\ln(1-3x) - 3 \cdot \frac{\cos(x)}{1-3x} \\
y' &= -y\left(\sin(x)\ln(1-3x) + 3 \cdot \frac{\cos(x)}{1-3x}\right) \\
y' &= -(1-3x)^{\cos(x)}\left(\sin(x)\ln(1-3x) + 3 \cdot \frac{\cos(x)}{1-3x}\right)
\end{aligned}$$

Exercise 0.3. Find the tangent line at the point $(1,1)$ for the graph of $x^2 + xy + y^2 = 3$. (3 marks)

Solution: (1 mark per item)

- Differentiation

$$\frac{d}{dx}(x^2 + xy + y^2) = \frac{d}{dx}(3)$$
$$2x + xy' + y + 2yy' = 0$$
$$y'(x + 2y) = -2x - y$$
$$y' = -\frac{2x + y}{x + 2y}$$

- Plugging (1,1) into y', we get $y' = -1$
- The equation of the tangent line at $(1, 1)$ is $y - 1 = -1(x - 1) \Longrightarrow y = -x + 2$

Exercise 0.4. Find the tangent line at the point $(2, 5)$ for the graph of $2x^2 + y^2 = 33$. (3 marks)
Solution: (1 mark per item)

- Differentiate

$$\frac{d}{dx}(2x^2 + y^2) = \frac{d(33)}{dx}$$
$$4x + 2yy' = 0$$

- Isolate y' and compute the slope at $(2, 5)$

$$y' = -\frac{4x}{2y} = -\frac{2x}{y}$$
$$y' = -\frac{4}{5}$$

- Write the equation

$$y - 5 = -\frac{4}{5}(x - 2)$$