# Implicit Differentiation.

- When we do not know the explicit. formula of the function. but we know the equation that the function obeys
- when we have a complicated explicit formula ~~for~~ of the function. ~~and~~ the function obeys a simple. equation:

<u>example 1</u>: Differentiate the following expression:

. $e^{f(x)} = x$.

$$\frac{d\left(e^{f(x)}\right)}{dx} = \frac{dx}{dx}.$$

Using the chain rule, we have.

$$\frac{de^{f(x)}}{dx} = e^{f(x)} . f'(x) \quad \text{and} \quad \frac{dx}{dx} = 1.$$

$$\Rightarrow e^{f(x)} . f'(x) = 1.$$

Know, we can solve for $f'(x)$.

$$\Rightarrow f'(x) = e^{-f(x)}$$

example 2 : Find the derivative of $\sin(y) + y^3 = 6 - x^3$

$$\frac{d}{dx}\left(\sin(y) + y^3\right) = \frac{d}{dx}\left(6 - x^3\right)$$

$$\cos y \cdot \frac{dy}{dx} + 3y^2 \cdot \frac{dy}{dx} = -3x^2$$

$$\frac{dy}{dx}\left(3y^2 + \cos y\right) = -3x^2.$$

$$\frac{dy}{dx} = -\frac{3x^2}{3y^2 + \cos y}.$$ derivative of $y(x)$ W.R.T. $x$.

But we can also compute the derivative of $x(y)$. (W.R.T $y$).

$$\frac{d}{dy}\left(\sin(y) + y^3\right) = \frac{d}{dy}\left(6 - x^3\right).$$

$$\cos y + 3y^2 = -3x^2 \cdot \frac{dx}{dy}.$$

$$\frac{dx}{dy} = \frac{-3x^2}{3y^2 + \cos y} \Rightarrow$$ derivative of $x(y)$ W.R.T $y$.

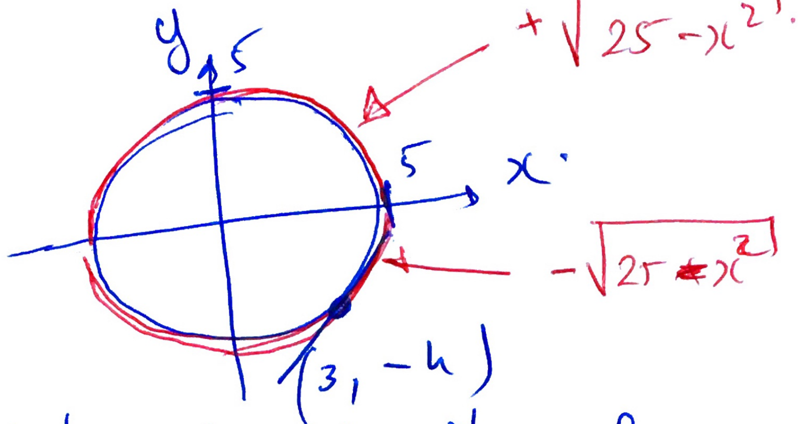
~~example 3: Differentiate~~ $\cancel{x^2 + y^2 = 25}$

$y = \sqrt{\ }$

- **Slope of a tangent line**

**example 3**: Find the slope of the line tangent to the graph of $x^2+y^2=25$ at the point $(3,-4)$.

It is a circle centered at $(0,0)$.




- What is the derivative of $y$?

$$x^2+y^2=25 \Rightarrow y=\pm\sqrt{25-x^2}$$

, since $(3,-4)$ lies on the bottom.

$$y=-\sqrt{25-x^2}$$

$$\Rightarrow y'=\frac{x^2}{\sqrt{25-x^2}} \quad \Rightarrow m=y'\Big|_{x=3}=\frac{3}{4}$$

Unfortunately, we can not do that for every equation. So, we use the implicit differentiation.

$$\frac{d}{dx}\left(x^2+y^2\right)=\frac{d}{dx}(25)$$

$$2x+2y\frac{dy}{dx}=0 \qquad\qquad \frac{dy}{dx}\equiv y'$$

$$2x+2yy'=0$$

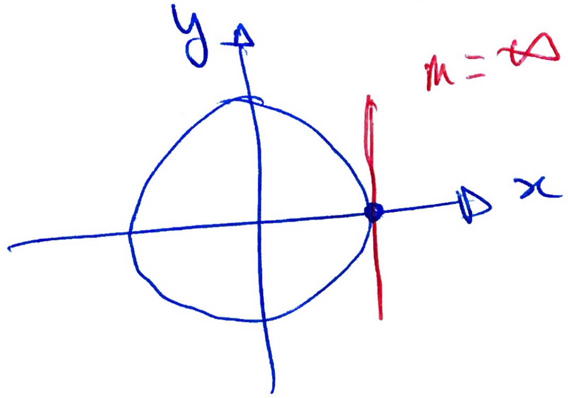
$$2yy' = -2x.$$

$$y' = -\frac{x}{y}$$

Plugging $(3,4)$ in the equation, we have:

$$m = y' = \frac{-(3)}{(-4)} = \frac{3}{4}$$

* At the point $(5,0)$, the slope is $\infty$ so the derivative $(y')$ does not exist.




example 4: Find the tangent lines at $x=1$ on the graph of $x^2 - y^2 - xy = 1$.

$x_0 = 1 \Rightarrow$ ~~$y_0$~~ $1 - y_0^2 - y_0 = 1$

$$y_0^2 + y_0 = 0.$$

$$y_0(1+y_0) = 0$$

$\Rightarrow x_0 = 1, \ y_0 = 0$

or $x_0 = 1, \ y_0 = -1.$