. Implicit differentiation (continue)

. Equations of normal lines to graphs of implicitly defined functions

Example1: $x^2+xy-y^2=1$. Find lines that are tangent and normal to the curve at the given point $(1,3)$

$$\frac{d}{dx}(x^2+xy-y^2)=\frac{d(1)}{dx}$$

$$2x+y+xy'-2y'y=0$$

$$y'(x-2y)=-2x-y.$$

$$y'=-\frac{2x+y}{x-2y}$$

* Tangent at $(1,3)$

$$y'=\frac{-2(1)-3}{1-2(3)}=\frac{-5}{-5}=1. \Rightarrow m=1.$$

$$y-y_1=m(x-x_1)$$

$$y-3=1(x-1)$$

$$\boxed{y=x+2}$$

* Normal at $(1,3)$ $\Rightarrow m=-1$

$$y-3=-1(x-1)$$

$y = -x + 1 + 3$ $\boxed{y = -x + 4}$.

- Power rule for rational exponents.

example 2: Find y' for the following:

$x^2 + y^2 = 9$.

$\frac{d}{dx}\left[x^2 + y^2\right] = \frac{d}{dx}(9)$

$2x + 2y'y = 0$

$2yy' = -2x$

$\boxed{y' = -\frac{x}{y}}$.

- Inverse properties of e^x and $\ln x$

Example 3: $e^{\cancel{2x+3y}}$ ~~$x+3xy$~~.

$e^{2x+3y} = x^2 - \ln(xy^3)$

$\frac{d}{dx}\left(e^{2x+3y}\right) = \frac{d}{dx}\left[x^2 - \ln(xy^3)\right]$

$\frac{d}{dx}(2x+3y)\, e^{2x+3y} = 2x - \left[\frac{y^3}{xy^3} + \frac{3xy^2y'}{xy^3}\right]$

$(2 + 3y')\, e^{2x+3y} = 2x - \frac{y^3 + 3xy^2y'}{xy^3}$

$$2e^{2x+3y} + 3y' e^{2x+3y} = 2x - \frac{1}{x} - \frac{3y'}{y}.$$

$$y'\left(3e^{2x+3y} + 3y^{-1}\right) = 2x - x^{-1} - 2e^{2x+3y}$$

$$\boxed{y' = \frac{2x - x^{-1} - 2e^{2x+3y}}{3e^{2x+2y} + 3y^{-1}}}$$

- Logarithmic functions

example 4: $y = \dfrac{x^5}{(1-10x)\sqrt{x^2+2}}$. Differentia

Can be done with quotient rule. but can be messy.

$$\ln y = \ln\left[\frac{x^5}{(1-10x)\sqrt{x^2+2}}\right]$$

$$\ln y = \ln(x^5) - \ln\left[(1-10x)\sqrt{x^2+2}\right].$$

$$= \ln(x^5) - \ln(1-10x) - \ln\left(\sqrt{x^2+2}\right).$$

$\Rightarrow \ln u = \dfrac{u'}{u}$

$$\frac{y'}{y} = \frac{5x^4}{x^5} - \frac{-10}{1-10x} - \frac{\frac{1}{2}(x^2+2)^{-\frac{1}{2}}\,2x}{(x^2+2)^{1/2}}$$

$$\frac{y'}{y} = \frac{5x^4}{x^5} + \frac{10}{1-10x} - \frac{x}{x^2+2} = \frac{5}{x} + \frac{10}{1-10x} - \frac{x}{x^2+2}$$

$$y' = y\left[\frac{5}{x} + \frac{10}{1-10x} - \frac{x}{x^2+2}\right]$$

$$\boxed{y' = \frac{x^5}{(1-10x)\left(\sqrt{x^2+2}\right)}\left[\frac{5}{x} + \frac{10}{1-10x} - \frac{x}{x^2+2}\right]}$$