Mean Value Theorem.

Suppose $f(x)$ is a function that satisfies both the following:

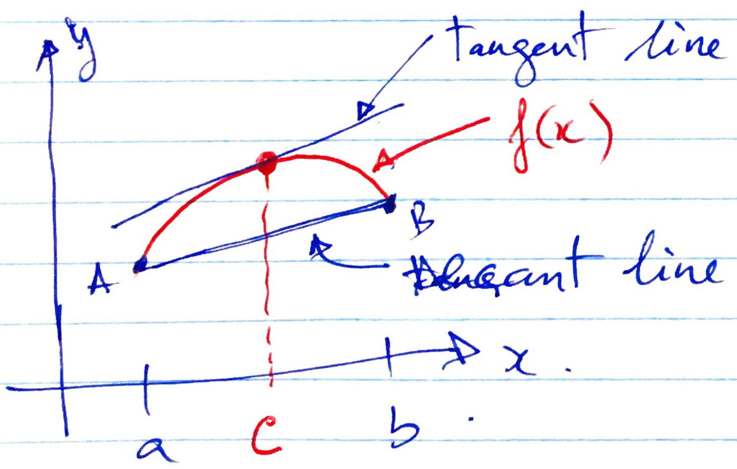
1. $f(x)$ is continuous on the closed $[a, b]$
2. $f(x)$ is differentiable on the open (a, b).

Then there is a number c such that $a < c < b$ and.

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

$$f(b) - f(a) = f'(c)\,(b - a).$$

Note that it does not tell where c is.



the tangent like and the secant line have the same slope.

Example 1: Determine all the numbers c which satisfy the conclusions of the Mean Theorem Value. for $f(x) = x^3 + 2x^2 - x$ on $[-1, 2]$

Solution:

- $f(x)$ is a polynomial. it is continuous and differentiable.

- $f'(x) = 3x^2 + 4x - 1$.

Now, to find the numbers that satisfy the conclusion of the Mean Value Theorem, we do:

$$f'(c) = \frac{f(2) - f(-1)}{2-(-1)} =$$

$$3c^2 + 4c - 1 = \frac{14-2}{3} = \frac{12}{3} = 4$$

$$\Rightarrow 3c^2 + 4c - 1 = 4$$

$$3c^2 + 4c - 5 = 0 \Rightarrow c = 0.7863 \text{ or } c = -2.1196.$$

Since the interval is $[-1, 2]$, $\boxed{c = 0.7863}$

- Inverse properties of exponential/logarithmic functions

Provided we are using the natural exponential function, we have:

$$f(x) = e^x \Rightarrow f'(x) = e^x.$$

For the general exponential function, we have:

$$f(x) = a^x \Rightarrow f'(x) = a^x \ln(a).$$

If $f(x)$ and $g(x)$ are inverse of each other,

$$g'(x) = \frac{1}{f'(g(x))}$$

So, if we have $f(x) = e^x$ and $g(x) = \ln x$, then:

$$g'(x) = \frac{1}{f'(g(x))} = \frac{1}{e^{g(x)}} = \frac{1}{e^{\ln x}} = \frac{1}{x}.$$

$$f'(x) = \frac{1}{g'(f(x))} = \frac{1}{\frac{1}{e^x}} = e^x$$

If we have $\log_a x = \frac{\ln x}{\ln a}$.

$$\frac{d}{dx}\left(\log_a x\right) = \frac{d}{dx}\left(\frac{\ln x}{\ln a}\right) = \frac{1}{\ln a}\frac{d}{dx}\left(\ln x\right)$$

$$\boxed{\frac{d}{dx}\left(\log_a x\right) = \frac{1}{x \ln a}.}$$