Math 104 section 108 Homework week 6 Solutions

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1 Price Elasticity of Demand

Exercise 1.1. A tailor is currently producing 80 suits per month and sells them for \$100 per suit. His monthly demand curve is given by $q = 100 - 2\sqrt{p}$. Find the current price elasticity of demand and use it to decide whether price should be raised or lowered to increase his revenue. (2.5 marks)
Solution: $q = 80, p = \$100$

$$q = 100 - 2\sqrt{p}$$

Find ε and decide to raise or lower the price.

$$\varepsilon = \frac{p}{q} \cdot \frac{dq}{dp} \qquad (0.5 \text{ mark})$$

$$\frac{dq}{dp} = \frac{-2}{2\sqrt{p}} = \frac{-1}{\sqrt{p}} \qquad (0.5 \text{ mark})$$

So,

$$\varepsilon = -\frac{1}{\sqrt{p}} \cdot \frac{p}{q} \qquad (0.5 \text{ mark})$$

Plugging $q = 80, p = \$100$, we have:

$$\varepsilon = -\frac{100}{10 \cdot 80} = -\frac{1}{8} \qquad (0.5 \text{ mark})$$

$|\varepsilon| < 1 \implies$ should increase the price. (0.5 mark)

Exercise 1.2. The price p (in dollars) and the demand q for a product are related by the following demand equation: $p^3 + q + q^3 = 38$. Find the elasticity of demand in terms of p and q for this product. (2.5 marks)
Solution:
Let's start by getting $\frac{dq}{dp}$

$$3p^2 + \frac{dq}{dp} + 3q^2\frac{dq}{dp} = 0 \qquad (0.5 \text{ mark})$$

$$\frac{dq}{dp} = \frac{-3p^2}{1+3q^2} \qquad (1 \text{ mark})$$

So, $\varepsilon = -\frac{3p^3}{(1+3q^2)q} \qquad (1 \text{ mark})$

2 Marginal Cost

Exercise 2.1. Suppose the demand curve for a product produced by a firm is given by $q = 270 - p$ and the cost function is $C(q) = 12q + \frac{4}{5}q^2$. Find the profit maximizing output for the firm. (2.5 marks)

Solution:
We will need to find the marginal revenue MR using the given demand function.
Recall that $R = pq$. If we isolate for p in the demand equation, we can multiply everything by q to find the revenue function.

$$p = 270 - q$$

$$R = 270q - q^2$$

This means that the revenue is:

$$\frac{dR}{dq} = 270 - 2q \quad \text{(0.5 mark)}$$

The marginal cost is:

$$\frac{dC}{dq} = 60 + 8q \quad \text{(0.5 mark)}$$

Setting $\frac{dR}{dq} = \frac{dC}{dq}$, we get:

$$270 - 2q = 60 + 8q$$

$$q = 21 \quad \text{(0.5 mark)}$$

How do we know this value of q gives us a maximum profit? We are interested often in the **break-even** points. That is, these are the points at which the revenues balance the costs and we make no profit. We want to understand these points since they help us decide our production behaviour since varying production when we are close to a break-even point can mean the difference between profitability and generating a loss. Mathematically, to find the break even points, we just set $R(q) = C(q)$ and solve for q. In this example, this gives

$$270q - q^2 = 12q + \frac{4}{5}q^2 \quad \text{(0.5 mark)}$$

$$258q = \frac{9}{5}q^2$$

$$q = 0 \quad \text{or} \quad q = 143 \quad \text{(0.5 mark)}$$

Hence, the more interesting break-even point is to produce $q = 143$. If we increase q from this value, we loose money.

3 Mean Value Theorem

Exercise 3.1. Suppose that we know that $f(x)$ is continuous and differentiable on $[6, 15]$. Let's also suppose that we know that $f(6) = -2$ and that we know that $f'(x) \leq 10$. What is the largest possible value for $f(15)$. (2.5 marks)

Solution:
Let's start with the conclusion of the Mean Value Theorem.

$$f(15) - f(6) = f'(c)(15 - 6) \quad \text{(0.5 mark)}$$

Plugging in for the known quantities and rewriting this a little gives,

$$f(15) - f(6) = f'(c)(15 - 6) = -2 + 9f'(c) \quad \text{(0.5 mark)}$$

Now we know that $f'(x) \leq 10$ so in particular we know that $f'(c) \leq 10$. This gives us the following,

$$\begin{aligned} f(15) &= -2 + 9f'(c) \quad \text{(0.5 mark)} \\ &\leq -2 + (9)10 \quad \text{(0.5 mark)} \\ &= 88 \quad \text{(0.5 mark)} \end{aligned}$$

All we did was replace $f'(c)$ with the largest possible value. This means that the largest possible value for $f(15)$ is 88.