Example: $f(t) = t(6-t)^{2/3}$ Find the inflection points and use the SDT to classify the critical points

Solution: $f'(t) = \dfrac{18-5t}{3(6-t)^{1/3}}$

$$f''(t) = \frac{-5\left[3(6-t)^{1/3}\right] - (18-5t)\left[\frac{1}{3}(-1)(6-t)^{-\frac{2}{3}}\right]}{9(6-t)^{4/3}}$$

$$f''(t) = \frac{10t-72}{9(6-t)^{4/3}}.$$

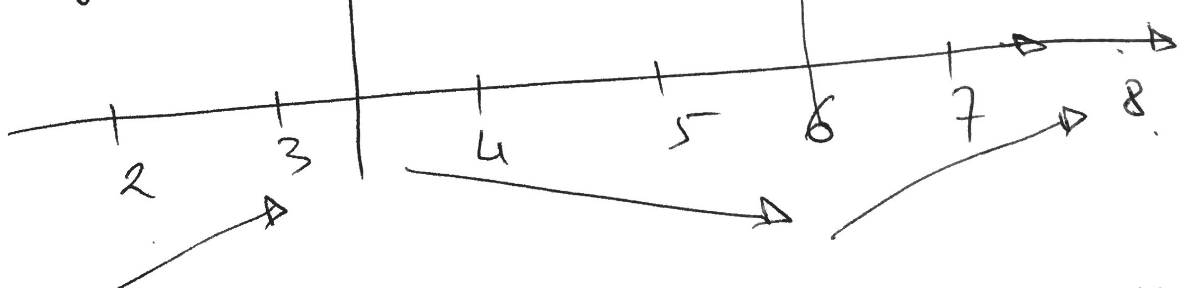
The critical points are. $t = \frac{18}{5} = 3.6$ $t = 6.$

Second Derivative. Test to Classify $t = 3.6$.

$f''(3.6) = -1.245 < 0 \Rightarrow$ relative maximum.

$f''(6)$?

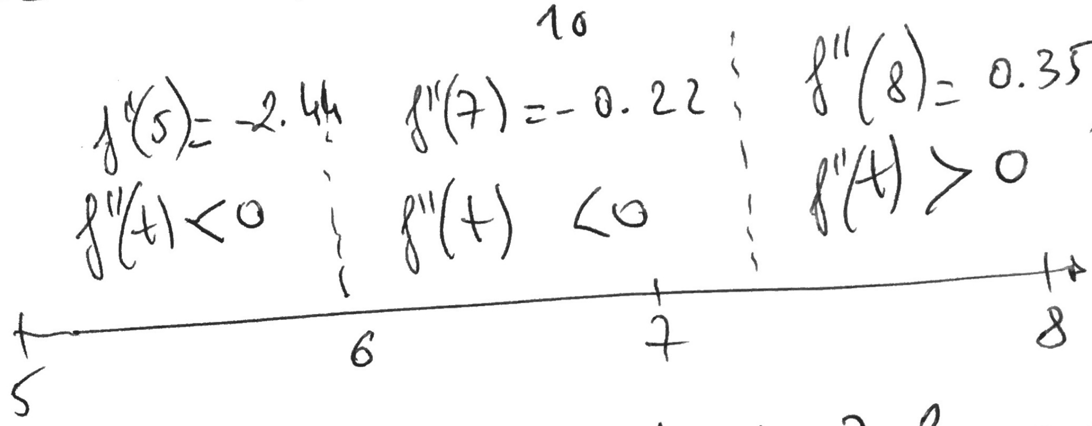
$f'(2) = 1.68$ | $f'(4) = -0.53$ | $f'(7) = 5.67$
$f'(t) > 0$ | $f'(t) < 0$ | $f'(t) > 0$.



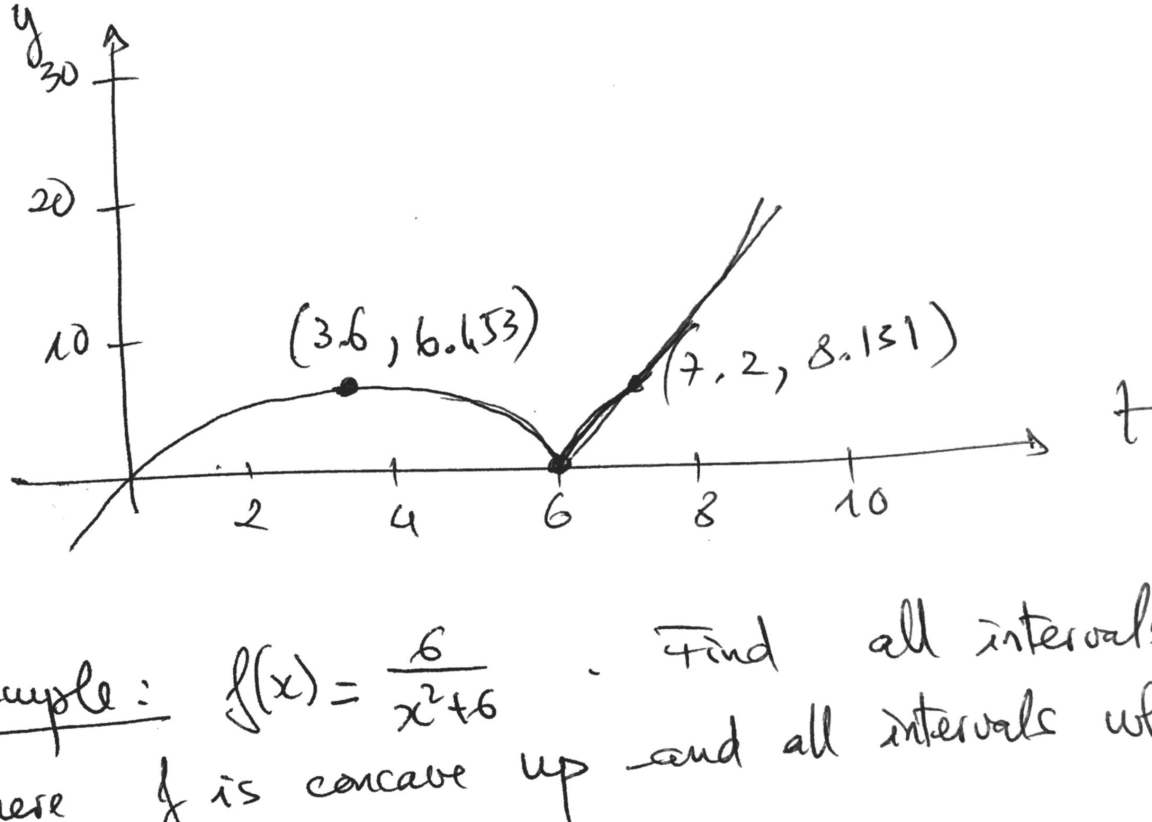
To finish the problem out, we will need the list of possible. inflection points.

①

$t = 6$ and $t = \frac{72}{10} = 7.2$.



So, the concavity changes at $t = 7.2$ and so this is the only inflection point for this function.



Example: $f(x) = \frac{6}{x^2+6}$. Find all intervals where f is concave up and all intervals where f is concave down.

Solution: $f'(x) = (-6)(x^2+3)^{-2}(2x) = -\frac{12x}{(x^2+3)^2}$

$$f''(x) = \frac{-12(x^2+3)^2 - (-12x)\left[2(x^2+3)2x\right]}{(x^2+3)^4}$$

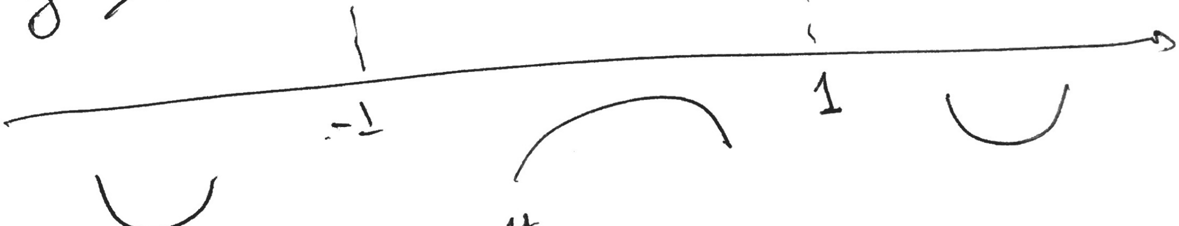
$$= \frac{-12(x^2+3)+48x^2}{(x^2+3)^3} = \frac{36x^2-36}{(x^2+3)^3}$$

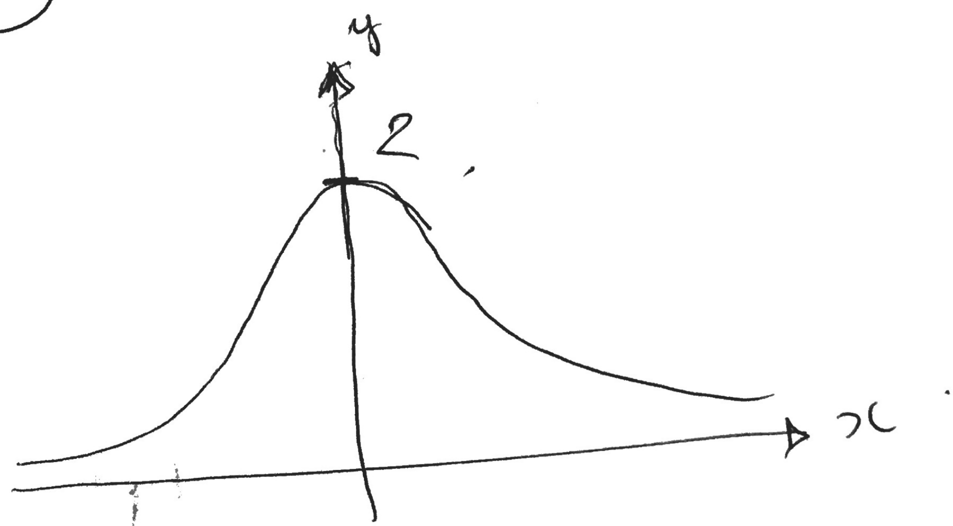
$$= \frac{36(x-1)(x+1)}{(x^2+3)^3}$$

$f''(x)=0 \Rightarrow x=1 \quad x=-1$

$f''(-2)=0.31$ $\quad$ $f''(0)=-1.33$ $\quad$ $f''(2)=0.31$

$f''>0$ $\quad$ $f''<0$ $\quad$ $f''>0$





Example: $g(x) = x^4 - 12x^2$. Where does g concave up and concave down.

$$g'(x) = 4x^3 - 24x$$

$$g''(x) = 12x^2 - 24$$

$$g''(x) = 0 \Rightarrow x^2 = 2 \quad x = \pm\sqrt{2}$$

Is the second derivative undefined anywhere? NO.

Now it's time to test the 3 regions.

concave up	concave down	concave up
$+$	$-$	$+$

$g''(-2) = 24 \quad -\sqrt{2} \quad g''(0) = -24 \quad \sqrt{2} \quad g''(2) = 24$

Because the concavity switched signs at the two zeros of g'', there are inflection points at these two x values.

$$g(-\sqrt{2}) = -20$$
$$g(\sqrt{2}) = -20$$

$\Rightarrow$ g is concave up from $-\infty$ to $(-\sqrt{2}, -20)$

g is concave ~~down~~ from $(\sqrt{2}, -20)$ to $+\infty$.

g is concave down between $(-\sqrt{2}, -20)$ and $(\sqrt{2}, -20)$