Vertical MFN's and Credit Card No-Surcharge Restraints: On-line Appendix

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Equilibrium for vMFN game, with linear demand

This appendix solves the duopoly vMFN game, for the case of linear demand:

- 2 upstream suppliers each decide simultaneously whether or not to impose a vertical restraint on downstream competitive retailers that requires each retailer to charge no more for the supplier's product than for the rival's product. This is a "vMFN" restraint.
- Then the suppliers set wholesale prices (w_1, w_2)
- Then retailers set retail prices (p_1, p_2)
- Retailers have zero costs. If neither supplier imposes the agreement, retailers just set retail prices equal to wholesale prices.
- If both suppliers set the agreement, retailers set a common retail price, p, equal to the average of the wholesale prices, $(w_1 + w_2)/2$.

For d in (0,1), define the following demand and profit functions:

$$q_1(p_1, p_2) = 1 - p_1 + dp_2$$

$$q_2(p_2, p_2) = 1 - p_2 + dp_1$$

This is a fully general representation of a symmetric linear demand system since the parameters other than d in a linear demand function $q_1(p_1, p_2) = a - bp_1 + dp_2$ can be normalized to 1 through the choice of quantity units and currency units.

Profit functions are given by

$$\pi_1(w_1, w_2) = q_1(p_1, p_2)w_1$$

$$\pi_2(w_1, w_2) = q_2(p_1, p_2)w_2$$

where the retail prices are functions of (w_1, w_2).

{0,0} pricing subgame:

This is simply a Bertrand game in which the wholesale prices (w_1, w_2) are passed through to the retail prices (p_1, p_2). That is, for this game: $w_i = p_i$, $i = 1, 2$.

$$R_1^{00}(w_2) = argmax_{w_1} \pi_1(w_1, w_2)$$

and symmetrically for $R_2^{00}(w_1)$.

We can verify by solving first-order condition that

$$R_1^{oo}(w_2) = \frac{1}{2} + \frac{d}{2} w_2$$

and similarly for R_2^{00}.

The (symmetric) equilibrium of this game is w_{00}^* defined as the solution in w to

$$R_1^{00}(w) = w$$

We can verify that

$$w_{00}^* = \frac{1}{2-d}$$

Define $\pi_{00}^* = \pi_1(w_{00}^*, w_{00}^*)$. We can verify that

$$\pi_{00}^* = \frac{1}{(2-d)^2} \tag{1}$$

This is the profit both firms get from the {0,0} game.

{1,1} pricing subgame:

For this game, retail prices are $p_1 = p_2 = (w_1 + w_2)/2$.

Define $p(w_1, w_2) = (w_1 + w_2)/2$

Define $\pi_1^{11}(w_1, w_2) = q_1[p(w_1, w_2), p(w_1, w_2)] \cdot w_1$

Define

$$R_1^{11}(w_2) = argmax_{w_1} \pi_1^{11}(w_1, w_2)$$

We can verify by solving the first-order condition that

$$R_1^{11}(w_2) = \frac{1}{(1-d)} - \frac{w_2}{2}$$

The equilibrium for this subgame is w_{11}^* defined as the solution to

$$R_1^{11}(w) = w$$

We can verify that

$$w_{11}^* = \frac{2}{3(1-d)}$$

Define $\pi_{11}^* = \pi_1(w_{11}^*, w_{11}^*)$. We can verify that

$$\pi_{11}^* = \frac{2}{9(1-d)} \tag{2}$$

Since the firms continue to compete in prices, rather than quantities, the move to complements in the $(1, 1)$ game is a move (when demand is linear) not just from substitutes to complements but a move to prices as *strategic substitutes*. This is in contrast to their relationship as strategic complements in the $(0, 0)$ game.[1]

Under the vMFN, the reaction curves are thus downward sloping. In other words, the greater a rival's wholesale price (and, therefore, the greater the common retail price), the less inclined a firm is to raise the common retail price even further through an increase in its own wholesale price. Figure A1 compares the subgame pricing equilibria of the $(0, 0)$ Bertrand game and the $(1, 1)$ vMFN game for the case of linear demand.

[1]Taking the case of linear demand, $q_1(p_1, p_2) = 1 - p_1 + dp_2$, the profit function for firm 1 in the $(1, 1)$ game becomes

$$\pi_1 = (w_1)[1 - (1-d)(\frac{w_1 + w_2}{2})]$$

From this, $\partial^2\pi_1/\partial w_1 \partial w_2 = -(1-d)/2 < 0$, demonstrating strategic complementarity. The move from strategic complementarity to strategic substitutes with the vertical restraint is parallel to the same effect in the duopoly platform competition model of Boik and Corts (2016).

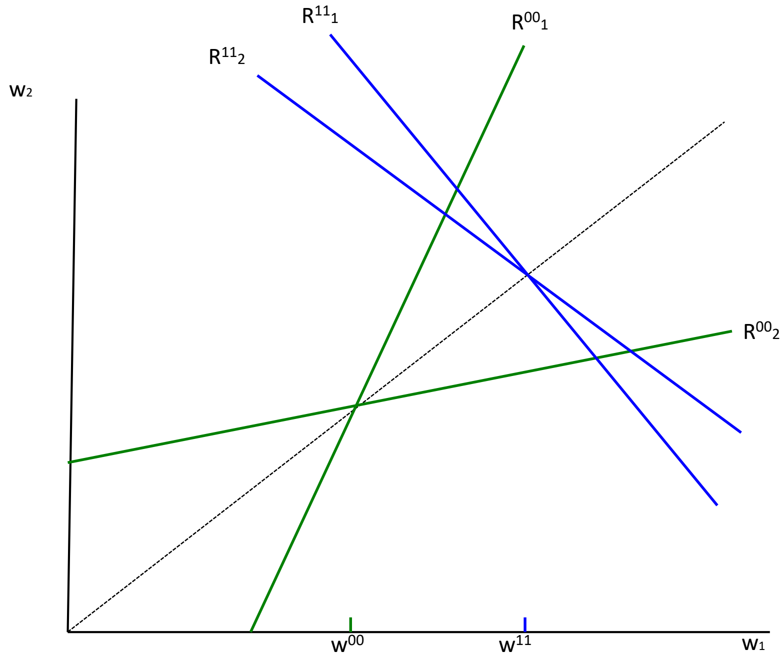

Figure A1: Reaction Curves for 00 and 11 Duopoly Pricing Subgames

This figure is a useful reference for solving the {1,0} game.

$\{1, 0\}$ **pricing subgame:**

This is the pricing subgame following decisions by Player 1 to impose the vMFN and player 2 to offer a contract without the vMFN. Consider player 1's reaction function. If player responds to a price w_2 by playing a price $w_1 < w_2$ then player 1's vMFN restraint is not binding. The players are (locally) in the $\{0, 0\}$game. On the other hand, if player 1 responds to w_2 by playing $w_1 > w_2$ then the restraint is binding, and the fact that player 2 has *not* imposed vMFN is irrelevant; the game is locally the same as the $\{1, 1\}$ game.

in fact the game is the same as the {1,1} game in which both players play the

Refering to Figure A1, the equilibrium in this game is the following:

- Player 2 plays the strategy $\hat{w}_2$ that makes player 1 indifferent between the following two strategies:
  - playing its best response for the {0,0} game, $R_1^{00}(\hat{w}_2)$, which is less than $\hat{w}_2$, earning payoff $\pi_1(R_1^{00}(\hat{w}_2), \hat{w}_2))$
  - playing its best response for the {1,1} game, $R_1^{11}(\hat{w}_2)$, which is greater than $\hat{w}_2$, earning payoff $\pi_1(R_1^{11}(\hat{w}_2), \hat{w}_2))$

- Player 1 plays the mixed strategy $\rho, (1-\rho)$ between $R_1^{00}(\hat{w}_2)$ and $R_1^{11}(\hat{w}_2)$ that makes $\hat{w}_2$ the best response to the mixed strategy. (One can verify that player 2's best response is decreasing in ρ.) This best response condition determines ρ.

- We will use shorthand $w_a = R_1^{00}(\hat{w}_2)$, and $w_b = R_1^{11}(\hat{w}_2)$ (**)

- The proof will make use of numerical calculations; with a single parameter, d, determining the game it is unnecessary to solve the game algebraically. We next define various functions recursively, on the basis of variables and functions already defined or determined.

- Define $\hat{w}_2$ as the solution in w_2 to the following equation

$$\pi_1(R_1^{00}(w_2), w_2) = \pi_1^{11}(R_1^{11}(w_2), w_2)$$

- Given $\hat{w}_2$, define w_a and w_b according to (**).

- Define $\hat{\rho}$ as the solution in ρ to:

$$\hat{w}_2 = argmax_{w_2}\{\rho\pi_2(w_a, w_2) + (1-\rho)\pi_2^{11}(w_b, w_2)\}$$

  All terms in this equation are either a calculated number, or a previously defined function, except for ρ. So the equation can be solved for ρ.

- We have determined, for any value of the parameter d, the players pricing strategies for the {1,0} game. Player 1 mixes over strategies w_a and w_b with probabilities ($\hat{\rho}$,(1-$\hat{\rho}$)). And player 2 plays $\hat{w}_2$.

- The final step for this subgame is to calculate the payoffs:

$$\pi_{10}^* = \pi_1(w_a, \hat{w}_2)$$

  where we are using the fact that player 1 is indifferent between the two points of support in its strategy. And for player 2:

$$\pi_{01}^* = \hat{\rho}\pi_2(w_a, \hat{w}_2) + (1-\hat{\rho})\pi_2^{11}(w_b, \hat{w}_2)\}$$

The Entire Game

- Having calculated the profits for each subgame, as functions of d, we plot the full set of payoffs for the pricing in Figure A2: $\pi_{00}^*, \pi_{10}^*, \pi_{01}^*, \pi_{11}^*$.

- The Figure demonstrates that $\pi_{10}^* > \pi_{00}^*$ and that $\pi_{11}^* > \pi_{01}^*$. This proves that given the payoffs from the pricing subgames in the simultaneous choices by the duopolists {adopt vMFN, do not adopt vMFN}, the strategy of adopting the vMFN is a dominant strategy.

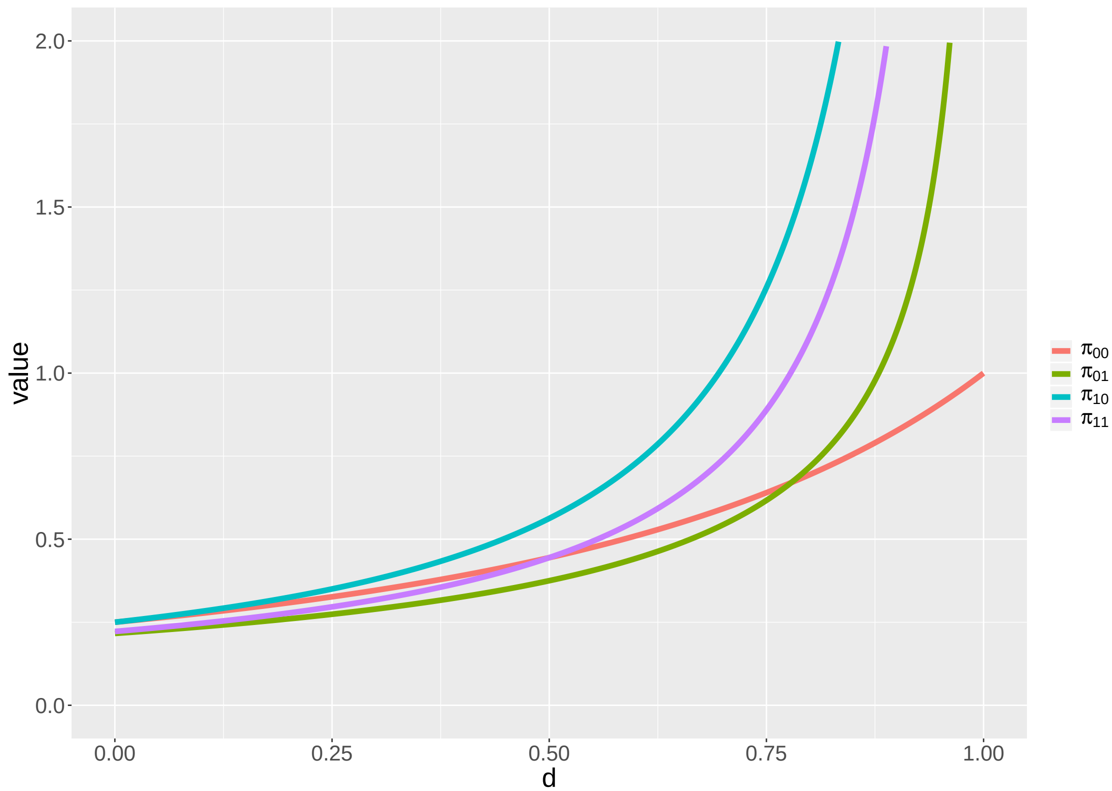

Figure A2: Payoffs $\pi_{00}^*, \pi_{01}^*, \pi_{10}^*, \pi_{11}^*$ as functions of d for symmetric linear demand

The fact that the vMFN is an individually dominant strategy leaves open the question of the impact on firms' profits of the joint adoption of the vMFN. Does a prisoners' dilemma arise, where the individually dominant strategies leave the firms both worse off? Comparing (1) and (2) we have:

$$\pi_{11}^* < \pi_{00}^* \quad \Leftrightarrow \quad \frac{2}{9(1-d)} < \frac{1}{(2-d)^2} \quad \Leftrightarrow \quad d < \frac{1}{2}$$

Thus, when $d < 1/2$, so that products are not close substitutes, the individual choices of vMFN or not yield a prisoner's dilemma. The intuition is clear for the case of independent products, $d = 0$. In this case, the diversion effect is zero so that the non-vMFN profits, π_{00}, are the collusive profits. The non-vMFN game raises price further because pricing choices are distorted by the cost-externalization effect.