Blog Integral Equations

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1. $\int_0^1 \frac{2x}{(x^2+5)^2-1}dx$ first we are going to use the substitution method
we substitute $u = x^2 + 5$
$du = 2xdx$
$dx = \frac{du}{2x}$
when $x = 0$ $u = 5$
when $x = 1$ $u = 6$
$\int_5^6 \frac{2x}{u^2-1}\frac{du}{2x}$
$\int_5^6 \frac{1}{u^2-1}du$
now we are going to use partial fraction to solve the rest
$\frac{1}{u^2-1}$
$\frac{1}{(u-1)(u+1)}$
$\frac{A}{u-1} + \frac{B}{u+1}$
$A(u+1) + B(u-1) = 1$
when $u = 1$
$A = \frac{1}{2}$
when $u = -1$
$B = -\frac{1}{2}$
$\frac{1}{2}\int_5^6 (\frac{1}{u-1} - \frac{1}{u+1})du$
$\frac{1}{2}(log(u-1) - log(u+1))|_5^6$
$\frac{1}{2}(log(5) - log(7) - (log(4) - log(6)))$
$\frac{1}{2}(log(5) - log(7) - log(4) + log(6))$
$\frac{1}{2}(log(\frac{15}{14})$

2.Evaluate $\displaystyle\int_0^1 t^{2n-1}(\cos(t^n) - \sin(t^n))dt.$

$$= \frac{1}{n}\int_0^1 t^n(nt^{n-1})(\cos(t^n) - \sin(t^n))dt$$

by integration by parts,

$$\int_0^1 t^n(nt^{n-1})(\cos(t^n) - \sin(t^n))dt = t^n(\sin(t^n) + \cos(t^n))|_0^1 - \int_0^1 t^{n-1}(\sin(t^n) + \cos(t^n))dt$$

by substitution $t^n = u$, $nt^{n-1}dt = du$, $t : 0 \to 1, u : 0 \to 1$

$$\int t^{n-1}(\sin(t^n)+\cos(t^n))dt = \frac{1}{n}\int_0^1 \sin u+\cos u du = \sin u-\cos u|_0^1 = \sin(1)-\cos(1)+1$$

Therefore,

$$\int_0^1 t^{2n-1}(\cos(t^n)-\sin(t^n))dt = \frac{1}{n}(t^n(\sin(t^n)+\cos(t^n))|_0^1 - \frac{1}{n}(\sin(1)-\cos(1)+1))$$

$$= \frac{1}{n^2}(n(\sin(1)+\cos(1)-1)-(\sin(1)-\cos(1)+1))$$

$$= \frac{1}{n^2}((n-1)(\sin(1))+(n+1)(\cos(1)-1))$$

3.$\int_0^1 (2x^3+2x)e^{x^2+1}dx$
$\int_0^1 (2x^3+2x)e^{x^2+1}dx = \int_0^1 2x(x^2+1)e^{x^2+1}dx$
Then we use the substitution and take $t = x^2+1$, $dt = 2x$.
$\int_0^1 2x(x^2+1)e^{x^2+1}dx = \int_1^2 te^t dt$
Then we calculate this by parts and take $u = t$, $du = dt$, $dv = e^t$ and $v = e^t$.
$\int_1^2 te^t dt = 2e^2 - e - \int_1^2 e^t dt = 2e^2 - e - e^2 + e = e^2$