A Review of the LISA and eLISA Projects

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LISA was to be a space-borne interferometer planned by the ESA and NASA. The project has been downsized to eLISA, which works using the same physics principles as its predecessor. The goal of the LISA-like projects is to detect gravitational waves at a lower range of frequencies than those measurable on Earth by stations like LIGO. While ground-based interferometers have proven successful in detecting ripples in spacetime, a space-born detector would escape the gravitational noise found on Earth. It could also measure smaller gravitational wave strains due to the possibility of longer interferometer arms. eLISA is a constellation of satellites rotating in an Earth-like "cartwheel" orbit around our sun. It works much like a regular interferometer, but requires corrections to account for special relativistic effects of the accelerating satellites.

I. INTRODUCTION

In 1916, Albert Einstein predicted the existence of gravitational waves after discovering that the field equations of general relativity exhibited wave solutions [1]. Later in 1981, Taylor and Wesiberg were able to indirectly demonstrate the existence of gravitational radiation by studying the orbital period decay of the binary pulsar PSR 1913+16 [2]. Just shy of a century since Einstein's postulate, the sensors of the advanced Laser Interferometer Gravitational-Wave Observatory (LIGO) were successful in the first direct detection of gravitational waves [1].

The idea of a space-borne large-scale interferometer has been around since the 1970s [3]. This was roughly when the concept of the Laser Interferometer Space Antenna (LISA) was born. The project was to be a cooperative achievement between the ESA and NASA. The main goal: to detect low frequency gravitational waves unaccessible from the surface of the Earth [4]. LISA was to be an equilateral triangle shaped interferometer formed by a constellation of satellites $5x10^6$ Km apart [7]. For reasons not discussed in this report, NASA dropped the joint project. Thus, LISA was eventually down sized to the evolved Laser Interferometer Space Antenna (eLISA) and is to be managed by the ESA alone. The physics behind this new LISA-like project is identical to that of its predecessor, with the exception of the interferometer arm length of $1x10^6$ Km [7]. For this reason, this report uses information from sources regarding LISA to explain eLISA.

This report will cover three main topics regarding the LISA-like projects. We will begin by touching on a few differences between eLISA and LIGO. The reason for this comparison is to explain to the reader why a space-borne interferometer is required if a ground-based one (i.e., LIGO) has already proven to be capable of detecting gravitational waves. Next, we will delve into the eLISA structure. In this section, we will further describe the eLISA satellites, the positioning of the satellite constellation in the solar system, and the eLISA orbit around the sun. Finally, we will explain how eLISA detects gravitational waves.

II. eLISA VERSUS LIGO

As mentioned before, the advanced LIGO installations have proven their ability to measure gravitational waves. We will now compare LIGO's characteristics with those theorized for eLISA.

First of all, space-borne interferometers can explore a much richer frequency range than their counterparts on Earth. To be more precise, eLISA will have the capability of taking gravitational radiation measurements of less than 1 Hz [7]. By contrast, LIGO can only perceive waves in the audio range from a few Hertz up to a couple of kiloHertz [3]. For reference, the gravity wave detected by LIGO rose from 35Hz up to 250 Hz [1].On Earth, there are unavoidable sources of gravitational noise in the form of moving objects, weather phenomena, and seismic motion. Therefore, only high frequency gravity waves will be distinguishable. Whereas in space, interferometers can escape these sources of noise.

Another important distinction between eLISA and LIGO is their measurable strain resolution. While LIGO can perceive gravitational wave strain of 10^{-19}, eLISA will register strains of 10^{-21} [7, 8]. This is equivalent to distinguishing a change in length of one-millionth the width of a proton. This difference is partially due to the dimension limitations on Earth. We cannot build an interferometer with arms $1x10^6$ Km long on the surface of our planet, this is greater than the circumference of the Earth. the LIGO interferometers in Hanford, Washington and Livingston, Louisiana are identical and have an arm length of just 4 Km [1]. Understandably, building dimensions are no obstacle in space. Here, we can theoretically make our interferometer as large as we want. The larger the interferometer, the smaller the strain we can detect.

III. eLISA STRUCTURE & CONFIGURATION

eLISA is planned to be a constellation of three satellites placed at the vertices of an equilateral triangle with $1x10^6$ Km sides. This section will cover general information regarding the three eLISA satellites, their position, and

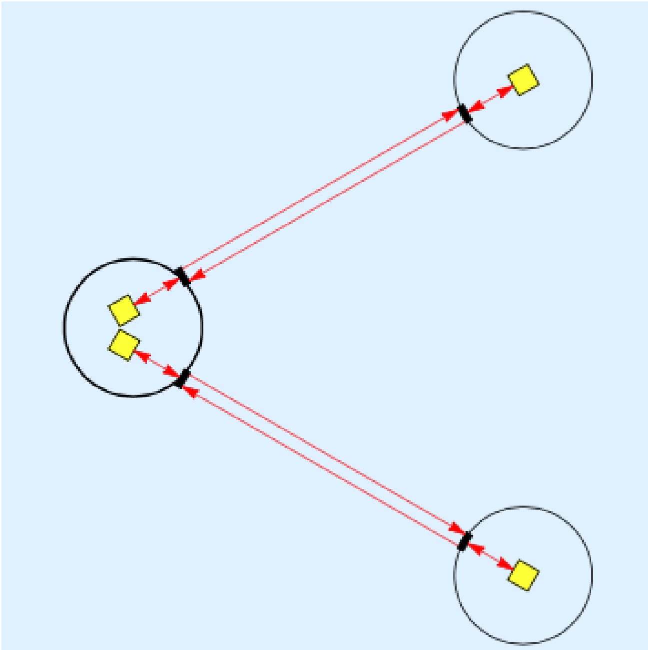

FIG. 1. Image provided by [4]. Diagram of LISA's satellites. The long single-headed arrows represent the laser signals forming the arms. The short double-headed arrows represent local interferometers within the satellites. The leftmost satellite is the mother satellite where the laser beams originate from

orbit in our solar system.

A. eLISA Satellites

One satellite, called the mother satellite [4], fires two laser signals which get reflected back by the satellites at the opposite vertex (see Fig. 1). Thus, forming the two interferometer arms, or links [4]. Unlike conventional Michelson interferometers, the mother satellite is not equipped with a beam splitter, but rather produces two independent phase-locked laser signals. The use of a beam splitter would be inefficient in the case of eLISA, as the diffraction of our light beam would make our signal inexistent by the time it reached the opposite end of the arm a million kilometres away [4]. When the signals return to the mother satellite, any phase shift is registered if existent. We will further develop the functioning of LISA and eLISA in section IV.

B. LISA Position in our Solar System

A diagram of the LISA satellites in orbit is shown on Fig.2. The positioning of eLISA will be identical. The entire configuration is placed 20^o behind our planet and follows an Earth like orbit around the sun [3]. This is roughly 50 million Km away from the Earth and we can

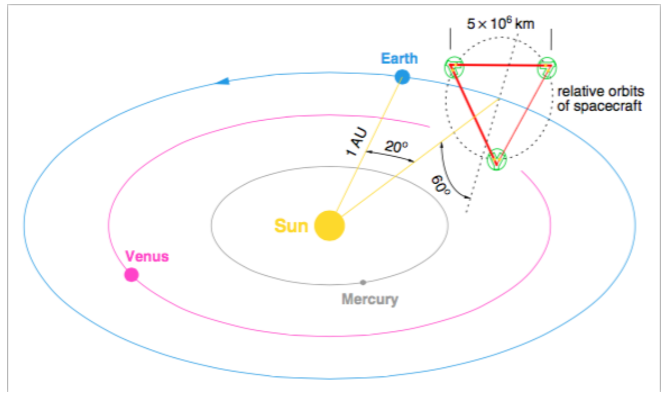

FIG. 2. Image provided by [3]. Diagram of LISA's position in our solar system.

allow the configuration to drift even further to about 70 million Km (the distance limit for comunication) [3, 7]. At these distances, the satellites are far enough to avoid Earth-based gravitational noise. As seen from Fig.2, the configuration also needs to be at a 60^o angle with respect to the orbital plane. We explain this parameter requirement in the following subsection.

C. eLISA Orbit

For a set of satellites to function as an interferometer, we require that each satellite maintain a constant distance with respect to the others. This is because each satellite pair is an interferometer arm and it would be useless if they changed length (i.e., without the perturbation of a gravitational wave). One would initially think of placing each satellite in parallel orbits around the sun. Although this might seem to work initially, the configuration would soon break off. This is because each satellite will try to follow its own orbit where the sun is directly in the centre. Therefore, the individual orbits will not be parallel, but will cross each other as the satellites follow their geodesic around the sun (see Fig.3).

This is why we require a 60^0 angle with respect to the orbital plane depicted on Fig.2. This initial configuration will allow each satellite to follow its proper orbit, while maintaining equal distance with respect to the two other satellites. As a consequence, eLISA will follow a "cartwheel" orbit [7]. That is, it will spin 360^0 around it's central points as it completes a solar cycle.

IV. FUNCTIONING OF eLISA

Before we explain how eLISA detects ripples in spacetime, we will give a brief theoretical explanation of the process.

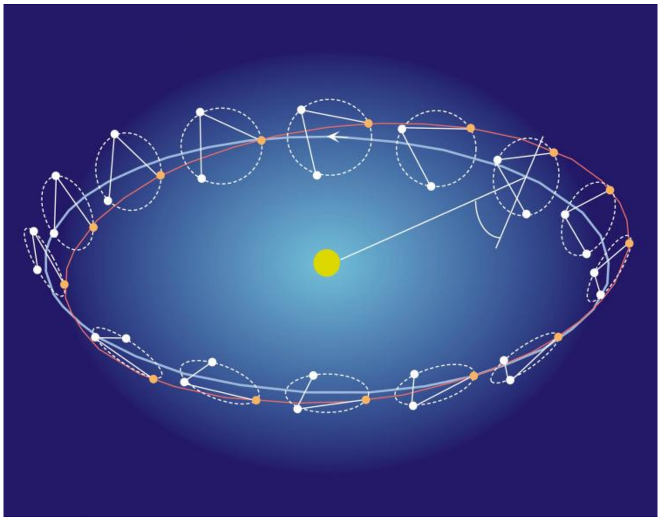

FIG. 3. Image provided by [7]. Diagram of eLISA's "cartwheel" orbit in the solar system. The orbit of a single satellite is shown with respect to the orbit of the central point of eLISA

A. Theoretical Detection of Gravity Waves

To detect a gravitational wave, we can monitor the relative distance between two tests masses in spacetime [6]. For example, say we have two stationary masses a distance L_* apart in the xy-plain in flat Minkowski spacetime given by:

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 \quad (1)$$

Both test masses will follow the straight line geodesic and the relative distance between the mass will be unperturbed. Lets consider what happens when a ripple in spacetime passes by in the z-direction. A metric is a symmetric, position dependant matrix that describes the general geometry of spacetime [6]. For flat spacetime, the metric is given by $\eta_{\alpha\beta}$ = diag(-1,1,1,1). However, the gravitational wave will bend space and time. Therefore, our metric will change to [6]:

$$g_{\alpha\beta}(t) = \eta_{\alpha\beta} + h_{\alpha\beta}(x) \quad (2)$$

Where $h_{\alpha\beta}(x)$ is the metric describing the perturbation in spacetime. In our example, it can be written as:

$$h_{\alpha\beta}(t,z) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} f(t-z) \quad (3)$$

Where the $f(t-z)$ is some function representing the fluctuations of the gravitational wave. Our spacetime will now be described by [6]:

$$dS^2 = -dt^2 + [1+f(t-z)]dx^2 + [1-f(t-z)]dy^2 + dz^2 \quad (4)$$

We must now measure the relative distance between the masses using the (4) rather than (1) [6]:

$$L(t) = \int_0^{L_*} \sqrt{1 + h_{xx}(t,0)}dx \approx L_*[1 + \frac{1}{2}h_{xx}(t,0)] \quad (5)$$

In (5) we assume one test mass to be at the origin while the other is on the x-axis. Lets generalize the second mass to be a distance L_* from the origin in the direction of $\hat{n}$ which lies on the xy-plane. We can then define the fractional strain produced by a gravitational wave as [6]:

$$\frac{\delta L(t)}{L_*} = \frac{1}{2}h_{ij}(t,0)n^i n^j \quad (6)$$

B. eLISA Method for Gravity Wave Detection

In practice, when a gravitational wave passes by eLISA, it will alternately increase and decrease the lengths of the interferometer arms by different amounts. This will cause a phase shift between the light signals travelling either arm and will be registered by the mother satellite upon return of the signals. In this sense, LISA-like interferometers work much the same as any interferometer on Earth. However, there is one more step of complexity we must account for with eLISA. Recall that the entire configuration is in orbit around the sun and is cartwheeling about its centre. Meaning there will be relativistic effects on the frequencies of the beams. This is where the local interferometers in each satellite come into play (see Fig. 3). The local interferometers are used to keep track of the inertial frames of reference of each satellite [4]. This way, eLISA can measure the acceleration of the receiving satellite with respect to the emitter or vice versa. Any changes in frequency due to special relativistic effects can be removed prior to checking for phase shifts. Thus, assuring any abnormalities in our signals are due to gravitational phenomena [4].

V. CONCLUSION

Gravitational waves are produced by astronomical level events (or at least the ones with large enough amplitudes to measure). Detecting and analyzing them could give us insight into the formation of supermassive black holes via binary black hole mergers, binary pulsars, the formation of galaxies, and possibly even give us clues about the early universe just seconds after the Big Bang [7, 8]. Thanks to LIGO, the existence of gravitational waves has been verified. For this reason, it is important for the scientific community to strive for better ways of detecting gravitational waves. eLISA, escaping the gravitational noise and dimension constraints on Earth, would be the ideal candidate project.

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