Exercises

1. National Income Formula: $Y = C + I + G + (X - M)$
2. Discounting: $PV = FV(1+r)^{-n}$
3. Compounding: $FV = PV(1+r)^{n}$
4. Bond Evaluation:

$$P = I\left[\frac{1-(1+r)^{-n}}{r}\right] + F(1+r)^{-n}$$

5. Probability:

$$\begin{aligned}
P(A \cup B) &= P(A) + P(B) - P(A \cap B)\\
P(A \cap B) &= P(A)P(B|A)\\
P(A|B) &= \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A')P(A')}\\
P(A_j|B) &= \frac{P(A_j)P(B|A_j)}{\sum P(A_i)P(B|A_i)}
\end{aligned}$$

6. Variance:

$$\begin{aligned}
&\mathrm{VAR}(X) = \sigma_X^2 = E[(X-\mu)^2]\\
&\mathrm{VAR}(X) = \sigma_X^2 = E(X^2) - [E(X)]^2\\
&\mathrm{VAR}(X) = \sum_{i=1}^{n}\left(p_i \cdot (x_i-\mu)^2\right) = \sum_{i=1}^{n}\left[p_i \cdot \left(x_i - \sum_{i=1}^{n} p_i x_i\right)^2\right]\\
&\mathrm{COV}(X) = \sigma_{XY} = E(X \cdot Y) - [E(X) - E(Y)]\\
&\mathrm{VAR}(aX) = a^2\sigma_X^2\\
&\mathrm{VAR}(aX + bY + cZ) = a^2\sigma_X^2 + b^2\sigma_Y^2 + c^2\sigma_Z^2 + 2ac\sigma_{XZ} + 2bc\sigma YZ
\end{aligned}$$

7. Epsilon Delta Definition of Limits:

$$\lim_{x \to c} f(x) = L \iff (\forall \varepsilon > 0, \exists \delta > 0, \forall x \in D, 0 < |x-c| < \delta \implies |f(x) - L| < \varepsilon)$$

8. Definition of the Derivative:

$$\begin{aligned}
\frac{\mathrm{d}f}{\mathrm{d}x}(x) &= \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}\\
\frac{\mathrm{d}f}{\mathrm{d}x}(c) &= \lim_{x \to c} \frac{f(x) - f(c)}{x - c}
\end{aligned}$$

9. Definition of the Integral:

$$\Delta x := \frac{x_f - x_0}{n}, n \in \mathbb{N}$$

$$\int_{x_0}^{x_f} f(x)\mathrm{d}x := \lim_{\Delta x \to 0} \sum_{i=1}^{n} f(x_i^*)\Delta x$$

$$\cdots$$

$$f(x) = \frac{\mathrm{d}F}{\mathrm{d}x}(x)$$

$$\implies \int_{x_0}^{x_f} f(x)\mathrm{d}x = F(x)|_{x_0}^{x_f}$$

10. Vectors:

$$\mathbf{a} + \mathbf{b} = \sum_{i}^{n} (a_i + b_i)\hat{\mathbf{e}}_i$$

$$\mathbf{a} \cdot \mathbf{b} = \sum_{i}^{n} a_i \cdot b_i$$

$$\mathbf{a} \times \mathbf{b} = (a_2 b_3 - a_3 b_2)\hat{\mathbf{i}} + (a_3 b_1 - a_1 b_3)\hat{\mathbf{j}} + (a_1 b_2 - a_2 b_1)\hat{\mathbf{k}}$$

$$\mathrm{proj}_{\mathbf{v}}\mathbf{a} = \frac{\mathbf{a} \cdot \mathbf{v}}{||\mathbf{v}||^2}\mathbf{v}$$

11. Static Equilibrium:

$$\sum \mathbf{F} = \mathbf{0}$$

$$\implies \sum F_x = \sum F_y = \sum F_z = 0$$

$$\sum \mathbf{M}_P = \mathbf{0}$$

$$\implies \left(\sum_i \mathbf{F}_i \times \mathbf{r}_i\right)_P = \sum_i \left(\det \begin{bmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ F_{i,x} & F_{i,y} & F_{i,z} \\ r_{i,x} & r_{i,y} & r_{i,z} \end{bmatrix}\right)_P = \mathbf{0}$$

12. Schrödinger Wave Equation:

$$\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x,t)\right]\Psi(x,t) = i\hbar\frac{\partial}{\partial t}\Psi(x,t)$$

13. Navier Stokes Equations:

$$\nabla \cdot \mathbf{V} = 0$$

$$\rho\left[\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla)\mathbf{V}\right] = -\nabla p + \nabla \cdot \tau + \rho \mathbf{g}$$

14. Bonus meme:

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