1. calculate $\displaystyle\int_0^1 x^5 e^{x^2} dx$

Let $u = x^2$, $\frac{du}{dx} = 2x$, $0^2 = 0$, and $1^2 = 1$

$$\int_0^1 x^5 e^{x^2} dx = \int_0^1 u^2 x e^u \frac{du}{2x} = \frac{1}{2}\int_0^1 u^2 e^u du$$

Let $v = u^2, v' = 2u, g = e^u$, and anti-derivative of $g = e^u$, then the function will be $\frac{1}{2}[u^2 e^u\big|_0^1 - \displaystyle\int_0^1 2ue^u]$

Let $k = 2u, k' = 2$, then the function will be $\frac{1}{2}[e - 2e + 2e - 2] = \frac{e}{2} - 1$

2.calculate $\int_0^3 (2x^3 + 18x)sin(x^2 + 9)dx$

$= \int_0^3 2x(x^2 + 9)sin(x^2 + 9)dx$

According to substitution method

let $w = x^2 + 9, dw = 2x$

$= \int_9^{18} w \cdot sin(w)dw$

According to integration by part method

let $u = w, du = dw, v' = sin(w)dw, v = -cos(w)$

$= -cos(w)w|_9^{18} - \int_9^{18} -cos(w)dw$

$= -cos(w)w|_9^{18} + sin(w)|_9^{18}$

$= sin(18) - sin(9) - 18cos(18) + 9cos(9)$

3. calculate $\displaystyle\int_0^{\frac{\pi}{2}} sinxlnsinxdx$

Solve: With integration by parts. Let u=$lnsinx$, $dv = sinxdx$

Then $du = cotxdx$, $v = -cosx$

So we have that $-ln(sinx)cosx|_0^{\frac{\pi}{2}} - \displaystyle\int_0^{\frac{\pi}{2}}(-cosxcotxdx)$=$-ln(sinx)cosx|_0^{\frac{\pi}{2}}$-$\displaystyle\int_0^{\frac{\pi}{2}} \frac{1 - sin^2x}{sinx}dx$

$\displaystyle\int_0^{\frac{\pi}{2}} \frac{1 - sin^2x}{sinx}dx$=$\displaystyle\int_0^{\frac{\pi}{2}} \frac{1}{sinx}$-$\displaystyle\int_0^{\frac{\pi}{2}} \frac{sin^2x}{sinx}$=$ln|tan\frac{\pi}{4}| - cos\frac{\pi}{2} - ln|tan0| + cos0$

=$ln(2)$-1