

Composition of Functions

Given $f(x) = x + \frac{1}{x}$ and $g(x) = \frac{x+1}{x+2}$, find a) $f \circ g$ b) $g \circ g$ and their domains,

a) $f(x) = x + \frac{1}{x}$

$$f(g(x)) = \frac{x+1}{x+2} + \frac{1}{\frac{x+1}{x+2}}$$

$$= \frac{x+1}{x+2} + \frac{x+2}{x+1}$$

$$= \frac{x+1}{x+2} \left(\frac{x+1}{x+1} \right) + \frac{x+2}{x+1} \left(\frac{x+2}{x+2} \right)$$

$$= \frac{(x+1)^2 + (x+2)^2}{(x+1)(x+2)}$$

$$= \frac{x^2 + 2x + 1 + x^2 + 4x + 4}{(x+1)(x+2)}$$

$$= \frac{2x^2 + 6x + 5}{(x+1)(x+2)}$$

Domain of $g(x)$: $x+2 \neq 0$
 $x \neq -2$

Domain $f(g(x))$: $x+1 \neq 0$
 $x \neq -1$
 $x+2 \neq 0$
 $x \neq -2$

Domain: $x \neq -1, -2,$



Composition of Functions (continued)

Given $f(x) = x + \frac{1}{x}$ and $g(x) = \frac{x+1}{x+2}$, find a) $f \circ g$ and b) $g \circ g$ and their domains.

$$a) f(g(x)) = \frac{2x^2 + 6x + 5}{(x+1)(x+2)} \quad \text{Domain: } x \neq \textcircled{-1}, -2$$

$$\frac{x^2 + 4x + 3}{(x+1)(x+2)} = \frac{\cancel{(x+1)}(x+3)}{\cancel{(x+1)}(x+2)}$$

$$b) g(x) = \frac{x+1}{x+2}$$

$$g(g(x)) = \frac{\frac{x+1}{x+2} + 1\left(\frac{x+2}{x+2}\right)}{\frac{x+1}{x+2} + 2\left(\frac{x+2}{x+2}\right)}$$

$$= \frac{\frac{x+1}{x+2} + \frac{x+2}{x+2}}{\frac{x+1}{x+2} + \frac{2x+4}{x+2}}$$

$$\Rightarrow = \frac{\frac{x+1+x+2}{\cancel{x+2}}}{\frac{x+1+2x+4}{\cancel{x+2}}}$$

$$= \frac{2x+3}{3x+5}$$

$$3x+5 \neq 0$$

$$3x \neq -5$$

$$x \neq -\frac{5}{3}$$

$$x+2 \neq 0$$

$$x \neq -2$$

$$\text{Domain: } x \neq -\frac{5}{3}, -2$$