Optimization problems may be done by using a four step technique which I outline below.

STEP 1

- 1. Draw a diagram. The diagram should be large and nearly drawn.
- 2. Label fixed quantities with their values. Label dimensions that vary with letters. Choose letters for any other items involved in the problem.

STEP 2

- 1. Define the quantity that you wish to maximize or minimize. Express this quantity as a function (note that it may be a function of several variables).
- 2. If the function is of two variables, you need to find a "constraint" so that you have a function of one variable.
- 3. Define the function you wish to optimize as a function of a *single variable*.

STEP 3

Differentiate your function from step 2 with respect to the variable you chose to express it in terms of. From here, find the critical points. That is, the values where the derivative is either 0 or where the derivative is undefined (but where the *function is defined*).

STEP 4

- 1. Use the first derivative test to determine whether the critical points determine local minima or maxima.
- 2. Note any restrictions on the domain of the function imposed by the problem, since many problems have "physical restrictions." Evaluate the function at the critical points which lie in the domain and at the endpoints of the domain (if they exist) to determine the global maximum or minimum.
- 3. If there are no endpoints; that is, the domain is not a closed interval, a minimum or a maximum may still exist elsewhere. So, you would need to provide an argument to support your conclusion on the global extrema.
- 4. Re-read the question to make sure you've answered the question and provided the quantity that was initially asked for. For example, if the question asks for the point at which a minimum occurs, just the x coordinate will not suffice.