Piece-Wise Functions

Def a piece-wise function $f(x)$ is made up of a family of subfunctions $f_i(x)$ so that the domains of each f_i, f_j do not disagree. We say

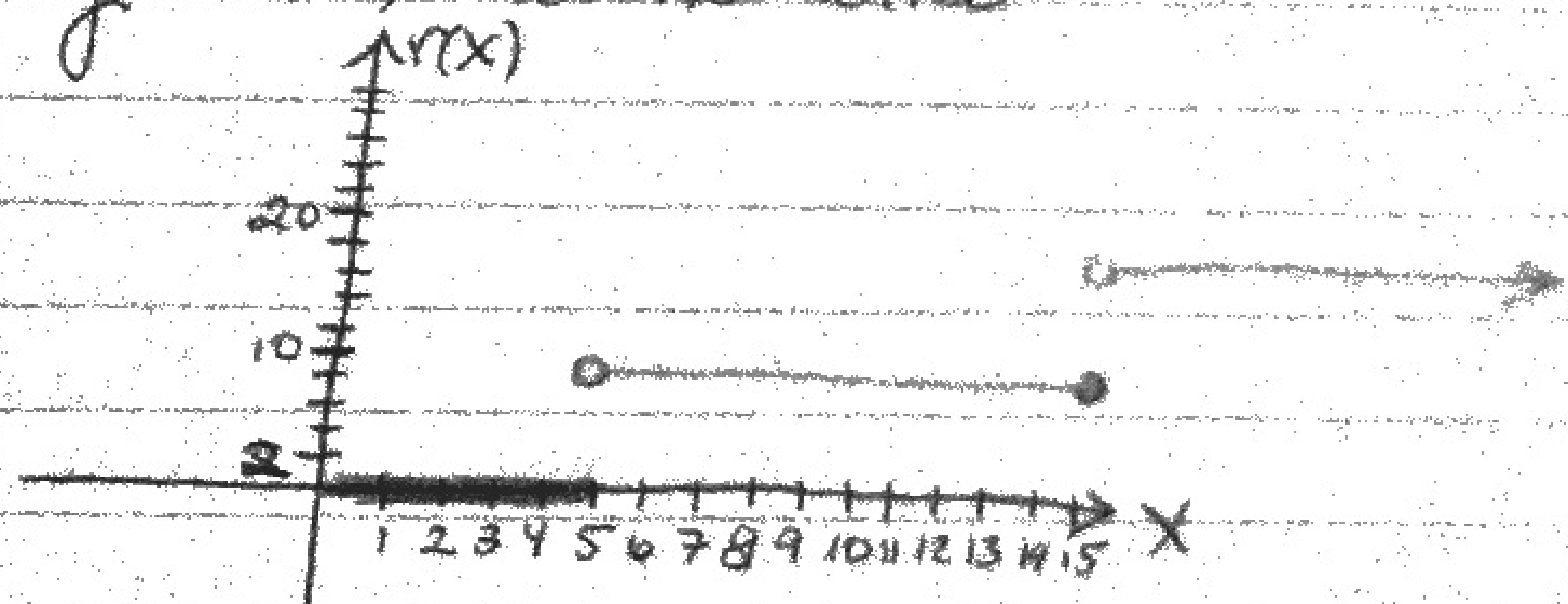
$$f(x) = \begin{cases} f_1(x) & , D_1 \\ f_2(x) & , D_2 \\ \vdots & \vdots \\ f_n(x) & , D_n \end{cases}$$

where D_i are the domains for each f_i

As a real life example of a piece-wise function, suppose a person changes the rate they charge for a service dependent on someone's age as follows.

$$r(x) = \begin{cases} 0, & \text{if age } 0-5, \ 5 \text{ included} \\ 10, & \text{if age } 5-15, \ 15 \text{ included} \\ 20, & \text{if age } 15 \text{ and up} \end{cases}$$

- The graph of $r(x)$ looks like



The domain of $r(x)$ is $[0, \infty) = \{x \in \mathbb{R} \mid 0 \leq x < \infty\}$
OR $0 \leq x < \infty$

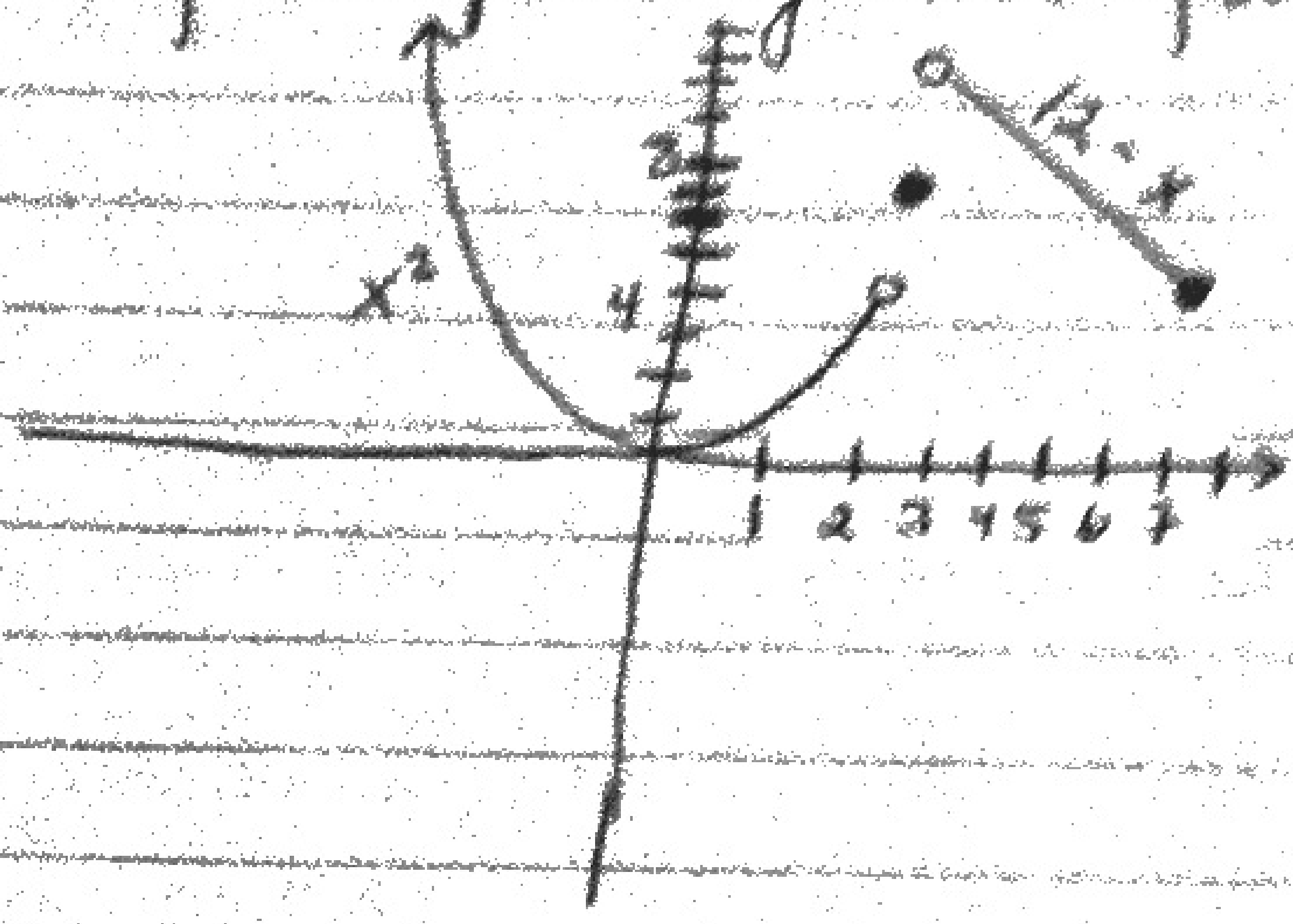
What is $f(4.99)$? Well $0 \leq 4.99 \leq 5$, so $f(4.99) = 0$

How about $f(8)$? Well, $5 < 8 \leq 15$, so $f(8) = 10$.

In formal notation, $r(x) = \begin{cases} 0, & 0 \le x \le 5 \\ 10, & 5 < x \le 15 \\ 20, & 15 < x < \infty \end{cases}$

Now, consider $f(x) = \begin{cases} x^2, & \text{if } x < 2 \\ 7.5, & \text{if } x = 2 \\ 12-x, & \text{if } 2 < x \le 7 \end{cases}$

Going from $-\infty$ and graphing, I work with the first part of the function $f(x)$



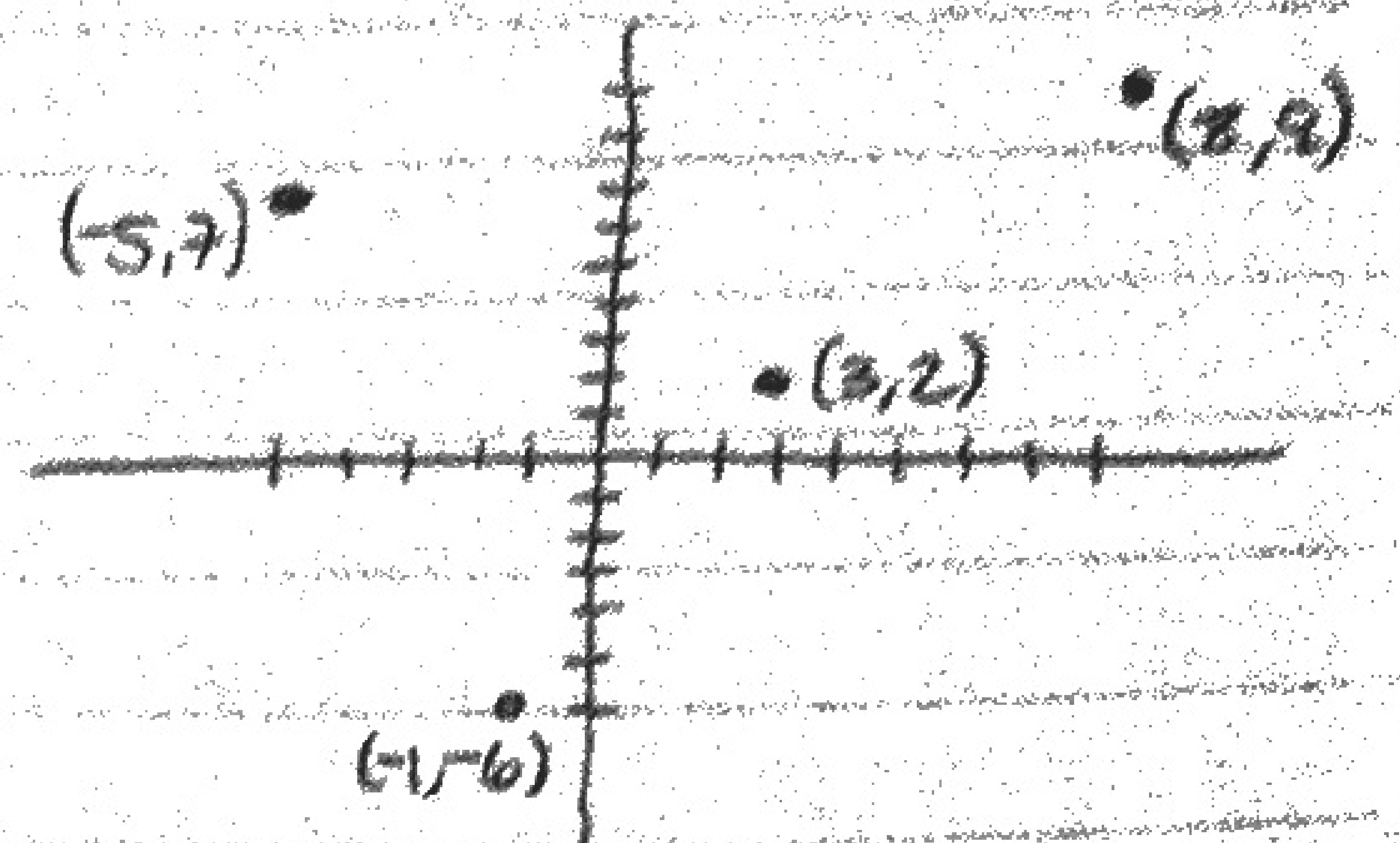
- From $-\infty$ to 2, everything is fine, until we hit $x = +2$
- At $x = +2$, we see x^2 is not defined there so we move to the function where $x = 2$ __is__ defined. In this case, it is $f(2) = 7.5$, a single point.

Similarly, at the next point in the domain, a number __VERY__ close to 2, we must move to the function $12-x$. As it is not defined there, we use an __open dot__. This function $12-x$ is defined up to $x = 7$, when $12 - 7 = 5$, so the point $(7,5)$ is on our graph.

What is $f(1.5)$? Well $1.5 < 2$ so we must choose x^2 and $f(1.5) = (1.5)^2 = 9/4$

* We can make up a function based on certain properties using piece wise functions.

Say I want a function where
$f(-5) = 7$,
$f(-1) = -6$,
$f(3) = 2$,
and $f(8) = 9$



Plotting these points, I have

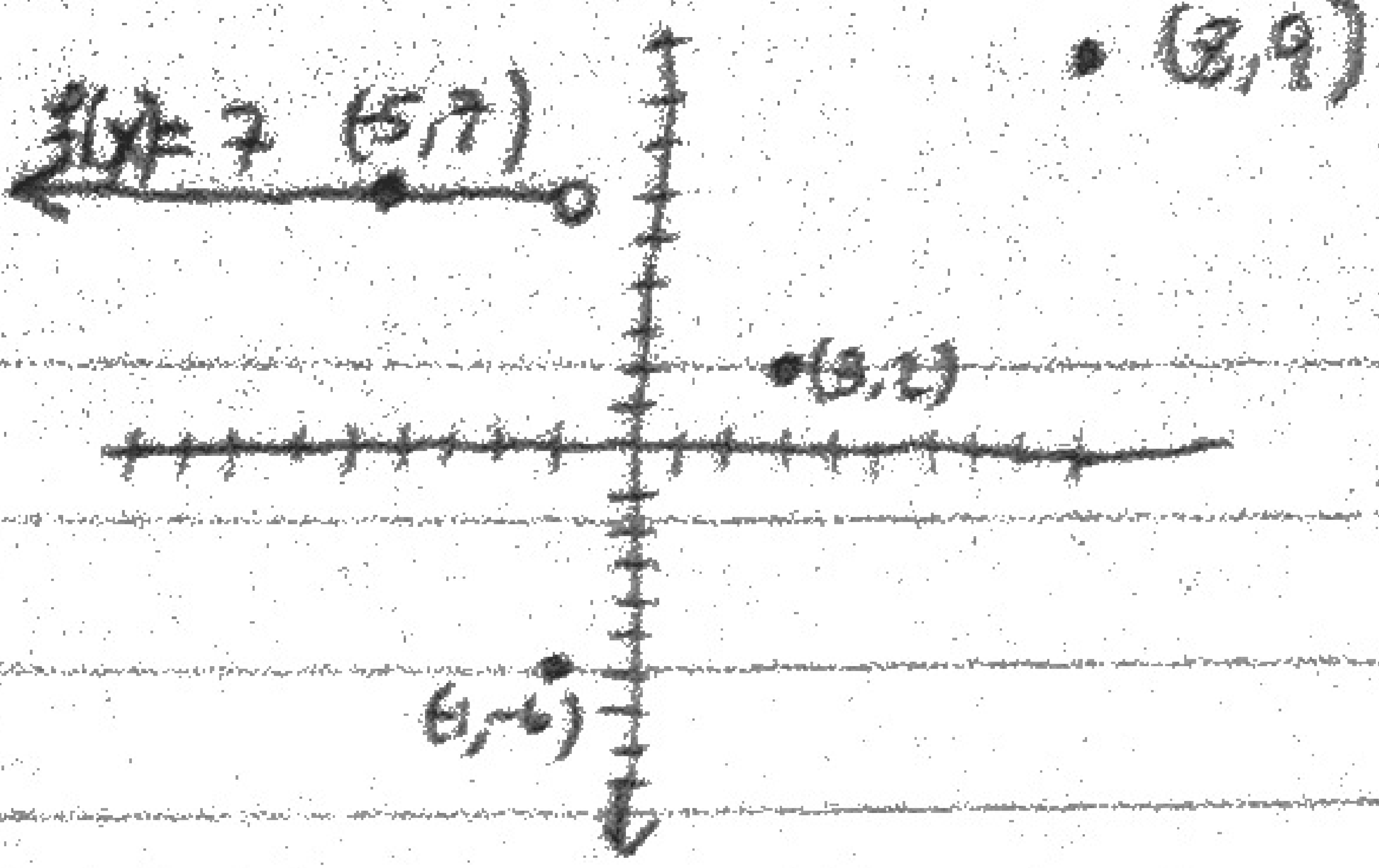
* At a glance, these points have no apparent pattern in mind.
* We could get fancy in making a piece-wise function, but lets go for simplicity.

* The simplest type of function is a "constant function" where $f(x) = c$, for some fixed #, c.
  - The graph, looks like a horizontal line.

* The problem numbers for our domain are $x = -5, -1, 3,$ and 8. So let's keep them in mind

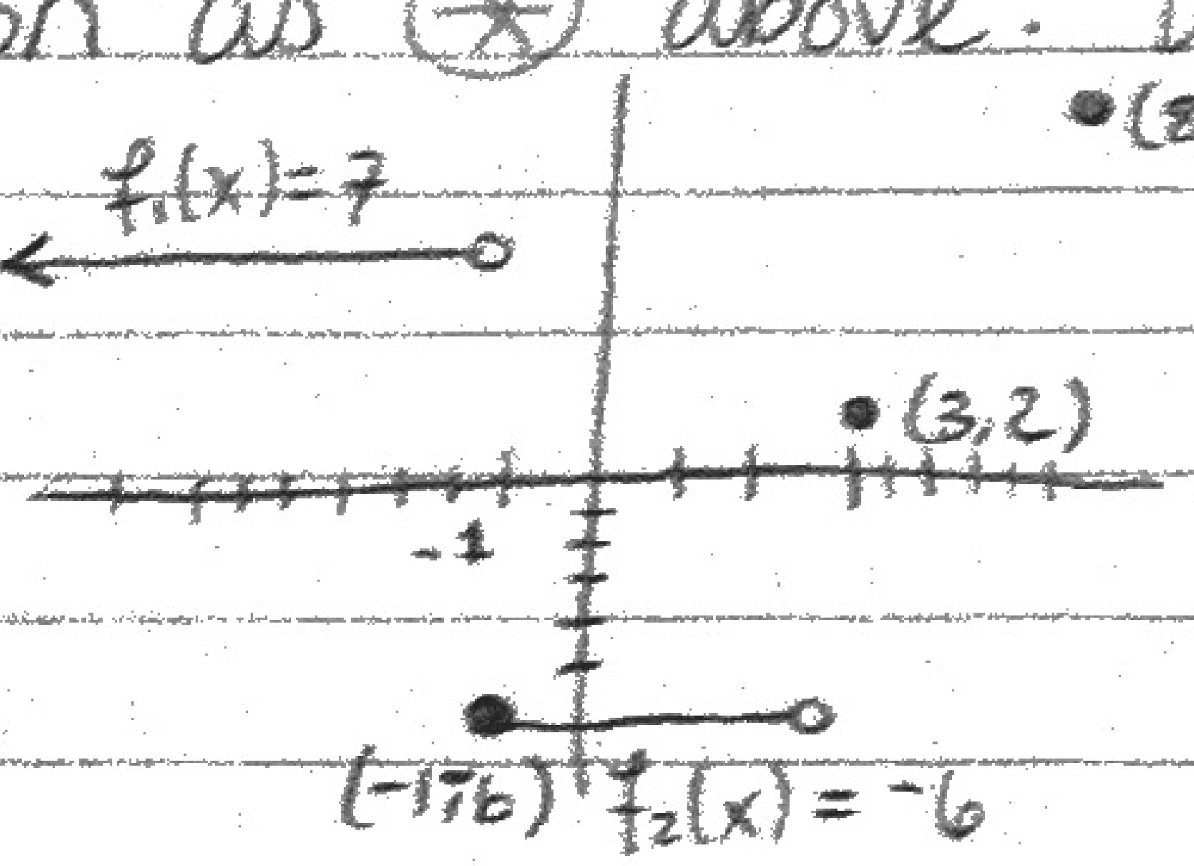
* Again, working from $-\infty$ to the right, I want to make sure the first function I choose contains $(-5, 7)$
  - How about $f_1(x) = 7$? $f_1(-5) = 7$, so it works
  - We will use this all the way to $x = -1$

So we have,

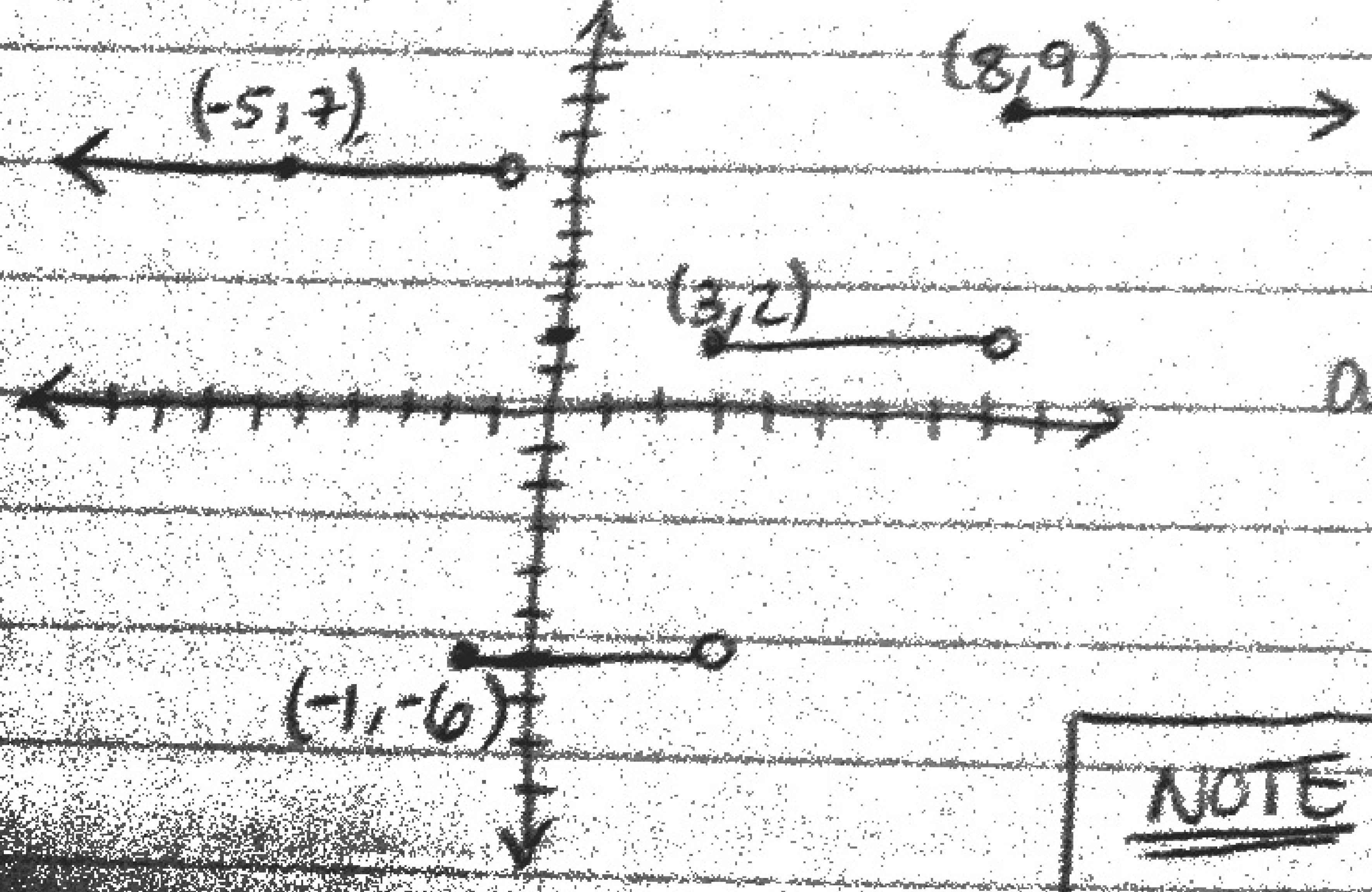


- We don't want a closed circle at (-1,7) because we KNOW we need a closed one at the point (-1,-6). So, we leave (-1,7) open.

- Do the same for the point (-1,-6). What function could we choose? Well, $f_2(x) = -6$ works since $f_2(-1) = -6$. Use this function until our next problem point $x = 3$, and put an open dot at (3,-6) for the same reason as (*) above. We get



Repeating the exact same process with $f_3(x) = 2$ and $f_4(x) = 9$, we have



and $f(x) = \begin{cases} 7, & x < -1 \\ -6, & -1 \leq x < 3 \\ 2, & 3 \leq x < 8 \\ 9, & x \geq 8 \end{cases}$

NOTE YOU COULD DO SOMETHING FANCY with parabolas & lines BUT WHY BOTHER. MAKE LIFE EA