Related Rates, October ~~XX~~ 27, ~~2016~~ 2017

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1. WARM-UP: Find y' for the curve $2x^2y^3 + 9yx^{10} = y^2(x + 10)$.

$\frac{d}{dx}\left(2x^2y^3 + 9yx^{10}\right) = \frac{d}{dx}\left(y^2(x+10)\right)$

$2x^2\,3y^2y' + y^3\,4x + 9y10x^9 + x^{10}\,9y' = y^2\cdot 1 + (x+10)2yy'$

$y'\left((2x^2\cdot 3y^2) + 9x^{10}\right) - 2y(x+10)y' = -4y^3x - 9y10x^9 + y^2$

$\therefore\ y' = \dfrac{-4y^3x - 90yx^9 + y^2}{6x^2y^2 + 9x^{10} - 2y(x+10)}$

2. A stone is dropped into a still pond and sends out a circular ripple whose radius increases at a rate of 3 feet per second. At what rate is the area enclosed by the ripple changing after 10 seconds?

$A = \pi r^2$ is the area of a circle. If area is changing with respect to time, we have $A(t) = \pi(r(t))^2$

We are given that radius changes at 3 ft/sec, so $r'(t) = 3$ ft/sec and $r(t) = 3t$

We want to find $A'(t)$ when $t = 10$ sec.

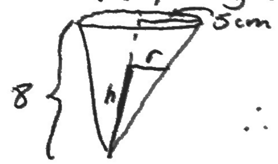
$A'(t) = \pi\cdot 2r(t)\cdot r'(t)$, so $A'(10) = \pi\cdot r(10)\cdot 3\,\text{ft/sec} = \pi(3\cdot 10)3\,\text{ft/sec} = 90\pi\ \frac{\text{ft}^2}{\text{s}}$

3. James Bond orders a martini. How fast is the height of the liquid inside the glass changing?

- The glass is a cone, where the thickness is negligible, diameter is 10cm, and height is 8cm.
- Liquid is being poured into the glass at a rate of $1\text{cm}^3/\text{sec}$.
- Want to know how fast the height is changing when the glass is 3/4 full.

*See solution to Chain Rule part one, October 25, 2017

* Key point here is the use of similar triangles to get the equation $V(t) = \frac{\pi}{3}(r(t))^2 h(t)$ down to one variable, being $h(t)$ in particular.



$\frac{h}{r} = \frac{8}{5}$

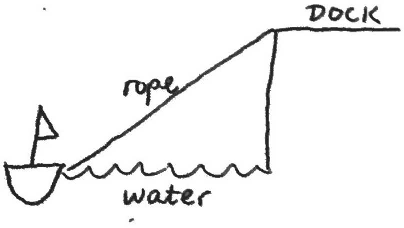
$\therefore\ r = \frac{5}{8}h$

$\Rightarrow V(t) = \frac{\pi}{3}\left(\frac{5}{8}h\right)^2\cdot h(t) = \frac{25\pi}{192}(h(t))^3$

$V'(t) = \frac{25\pi}{192}\cdot 3(h(t))^2\cdot h'(t)$

4. (a) A boat is drawn close to a dock by pulling in a rope as shown. How is the rate at which the rope is pulling in related to the rate at which the boat approaches the dock? Constant multiple, equal, or depends?

(b) In the same situation, suppose that the rope is being pulled at a constant rate. **True or False:** the closer the boat is, the faster it moves.



(a) Let x be the distance from the boat to the dock and z is the length of the rope. The height of the dock is constant, so let it be c. By Pythagoras, $x^2 + c^2 = z^2$

Using implicit diff. $2x\frac{dx}{dt} = 2z\frac{dz}{dt}$, so, $x\frac{dx}{dt} = z\frac{dz}{dt}$

So, $\frac{dx}{dt} = \frac{z}{x}\frac{dz}{dt}$. It depends how close the boat is to the dock

(b) As $x \to 0$, $\frac{dx}{dt}$ increases because $\frac{dz}{dt}$ has to be held constant by assumption