The First Derivative Test, November 10, 2017

Find and identify local minima and maxima by identifying intervals of increase and decrease of the following functions.

1. $f(x) = \dfrac{2x^2}{x^2 - 1}$

$f'(x) = \dfrac{-4x}{(x^2-1)^2}$ so CPs when $f'(x)=0$ or $f'(x)$ DNE

$-4x = 0$ $\quad$ $x^2-1=0$

$x=0$ $\quad$ $x = \pm 1$

	$x<-1$	$-1<x<0$	$0<x<1$	$x<1$
$-4x$	+	+	−	−
$(x^2-1)^2$	+	+	+	+
$f'(x)$	+	+	−	−
	inc	inc	dec	dec

- So, since $f'(0)=0$, we have a local max at $(0, f(0)) = (0,0)$
- What is happening at ± 1?
- $\lim_{x\to 1^-} f(x) = -\infty$ $\quad$ $\lim_{x\to 1^+} f(x) = +\infty$

$\lim_{x\to -1^+} f(x) = -\infty$ $\quad$ $\lim_{x\to -1^-} f(x) = \infty$

Also, $\lim_{x\to\infty} \dfrac{2x^2}{x^2-1} = 2$ and $\lim_{x\to-\infty} \dfrac{2x^2}{x^2-1} = 2$

2. $f(x) = \dfrac{e^x}{1+e^x}$

$f'(x) = \dfrac{e^x}{(1+e^x)^2}$ so $f(x)$ has no critical points.

$f'(x) > 0$ for all x, so $f(x)$ is increasing for all x.

What happens for $x \to \pm\infty$? $\lim_{x\to\infty} \dfrac{e^x}{1+e^x} = 1$ and $\lim_{x\to-\infty} \dfrac{e^x}{1+e^x} = 0$.

3. $f(x) = \dfrac{\tan(x)}{x}$ on $\left[\dfrac{-5\pi}{2}, \dfrac{5\pi}{2}\right]$

First, we note that something weird is happening at $x=0$.

$\lim_{x\to 0} \dfrac{\tan(x)}{x} = \lim_{x\to 0} \dfrac{\sin(x)}{x}\dfrac{1}{\cos(x)} = 1$

We need critical points. $f'(x) = \dfrac{x\sec^2 x - \tan(x)}{x^2}$. $f'(x)=0$ when

$x\sec^2 x - \tan x = 0 \Rightarrow x = \sin(x)\cos(x) \Leftrightarrow x=0$. Also, $f'(x)$ DNE

when $\sec(x)$ is not defined, which is when $x = \frac{2n+1}{2}\pi$, $n\in\mathbb{Z}$

So, we set up the table.

	$-\frac{5\pi}{2}<x<-\frac{3\pi}{2}$	$-\frac{3\pi}{2}<x<-\frac{\pi}{2}$	$-\frac{\pi}{2}<x<0$	$0<x<\pi/2$	$\pi/2<x<3\pi/2$	$3\pi/2<x<5\pi/2$
$x\sec^2 x - \tan x$	−	−	−	+	+	+
x^2	+	+	+	+	+	+
$f'(x)$	−	−	−	+	+	+