Trig and Exponential, November 3, 2017

1. WARM-UP: Determine the derivative of $\tan(x)$.

$$\frac{d}{dx}(\tan(x)) = \frac{d}{dx}\left(\frac{\sin(x)}{\cos(x)}\right) = \frac{(\sin(x))'\cos(x) - \sin(x)(\cos(x))'}{\cos^2(x)} = \frac{(\sin(x))^2+(\cos(x))^2}{\cos^2(x)}$$

$$= \frac{1}{\cos^2(x)} = \sec^2(x)$$

2. Fill in the following table with important derivatives:

f(x)	f'(x)	Domain
e^x	e^x	$\mathbb{R}$
$\log(x)$	$\frac{1}{x}$	$(0,\infty)$
$\sin(x)$	$\cos(x)$	$\mathbb{R}$
$\cos(x)$	$-\sin(x)$	$\mathbb{R}$
$\tan(x)$	$\frac{1}{\cos^2(x)} = \sec^2(x)$	$x \neq \frac{\pi}{2}+\pi n, n\in\mathbb{Z}$
$\sec(x) = \frac{1}{\cos(x)}$	$\sec(x)\tan(x)$	$x \neq \frac{\pi}{2}+\pi n, n\in\mathbb{Z}$
$\csc(x) = \frac{1}{\sin(x)}$	$-\csc(x)\cot(x)$	$x \neq n\pi, n\in\mathbb{Z}$
$\cot(x) = \frac{1}{\tan(x)}$	$-\csc^2(x)$	$x \neq n\pi, n\in\mathbb{Z}$
$\arctan(x)$	$\frac{1}{1+x^2}$	$(-\pi/2, \pi/2)$ (for now)

3. The half-life of Radium-226 is 1590 years. Suppose a sample of Radium-226 is 100 grams. Determine the formula for the mass after t years. How much mass is there after 1000 years?

- $m(t) = m(0)e^{kt}$ is the mass equation. We know $m(0)=100$, so all we need is k. We know $m(1590) = \frac{1}{2}m(0) = 50 = e^{k1590}100$

So, $e^{k1590} = \frac{1}{2}$ and $k = \frac{-\log(2)}{1590}$. Hence, $m(t) = 100e^{\frac{-\log(2)}{1590}t}$

and $m(1000) = 100e^{\frac{-\log(2)}{1.59}}$

4. Determine the derivative of $f(x) = x^x$.

$$f(x) = x^x \Leftrightarrow \log(f(x)) = \log(x^x) = x\log(x)$$

$$\frac{d}{dx}(\log(f(x))) = \frac{d}{dx}\, x\log(x)$$

$$\frac{f'(x)}{f(x)} = x\cdot\frac{1}{x} + \log(x)$$

$$\therefore\ f'(x) = x^x(1+\log(x))$$

5. Determine $\frac{d}{dx}(1-2x)^{\cos(x)}$

$$f(x) = (1-2x)^{\cos(x)} \Leftrightarrow \log(f(x)) = \log((1-2x)^{\cos(x)}) = \cos(x)\log(1-2x)$$

$$\frac{d}{dx}\log(f(x)) = \frac{d}{dx}(\log(1-2x)^{\cos(x)})$$

$$f'(x) = (1-2x)^{\cos(x)}\left[\cos(x)\frac{-2}{1-2x} + \log(1-2x)(-\sin(x))\right]$$

6. Determine $\frac{d}{dx}a^x$, where $a > 0$

$$y = a^x \Leftrightarrow \log(y) = x\log(a)$$

$$\frac{d}{dx}\log(y) = \frac{d}{dx}(x\log(a))$$ [note that $\log(a)$ is a constant!]

$$\frac{y'}{y} = \log(a)$$

$$y' = a^x \cdot \log(a).$$