Curve Sketching, November 17, 2017

1. Consider $f(x) = xe^x$.

(a) Find intervals of increase and decrease

$f'(x) = xe^x + 1\cdot e^x = e^x(x+1)$ Critical points: $f'(x) = 0$ or DNE

$f'(x)$ exists for all x, but $f'(x) = 0$ when $x = -1$

	$(-\infty,-1)$	$(-1,\infty)$
e^x	+	+
$(x+1)$	−	+
$f'(x)$	−	+
inc/dec	dec	inc

So, $f(x)$ is decreasing on $(-\infty,-1)$ and increasing on $(-1,\infty)$

(b) Identify local extrema

$(-1, f(-1)) = (-1, -e^{-1}) = (1, -1/e)$ is a local minimum

It will be a global min if we check behaviour to $\pm\infty$.

(c) Find intervals of concavity

$f''(x) = e^x(x+1) + 1(e^x) = e^x(x+2)$

Possible inflection points where $f''(x) = 0$ or DNE. $f''(x)$ always exists but $f''(x) = 0$ when $x = -2$.

	$(-\infty,-2)$	$(-2,\infty)$
$f''(x)$	−	+
CCU/CCD	CCD	CCU

So, $f(x)$ is concave down on $(-\infty,-2)$ and concave up on $(-2,\infty)$

(d) Find inflection points

$(-2, f(-2)) = (-2, \frac{-2}{e^2})$ is an inflection point of $f(x)$ since we change from CCD to CCU

(e) Find horizontal asymptotes

can apply l'hôpital's since $\lim_{x\to-\infty} x = \lim_{x\to-\infty} e^{-x} = \pm\infty$

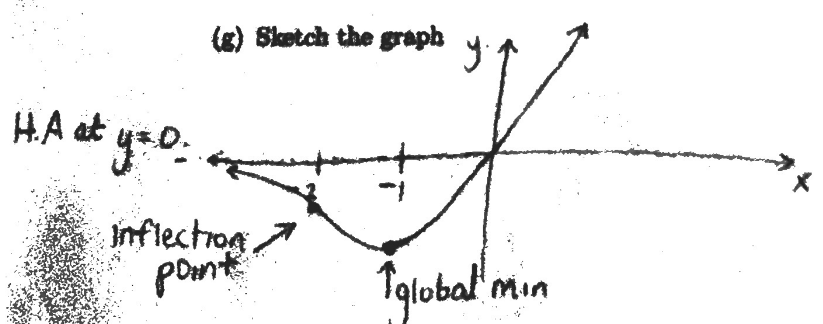
$\lim_{x\to\infty} xe^x = \infty$

$\lim_{x\to-\infty} xe^x = \lim_{x\to-\infty} \frac{x}{1/e^x} = \lim_{x\to-\infty} \frac{x}{e^{-x}} = \lim_{x\to-\infty} \frac{1}{-e^{-x}} = 0$

(f) Find vertical asymptotoes

No vertical asymptotes since $f(x)$ is defined for all x and continuous for all x.

(g) Sketch the graph



2. Consider $f(x) = \frac{log(x)}{x^2}$. DOMAIN: $x > 0$

(a) Find intervals of increase and decrease

$$f'(x) = \frac{x^2 \frac{1}{x} - \log(x) \cdot 2x}{x^4} = \frac{x(1-2\log(x))}{x^4} = \frac{1-2\log(x)}{x^3}$$

$f'(x)$ DNE when $x=0$ $f'(x)=0$ when $1-2\log(x)=0 \Rightarrow \log(x)=\frac{1}{2} \Rightarrow x=e^{1/2}$

	$(0, e^{1/2})$	$(e^{1/2}, \infty)$
$f'(x)$	+	−
inc/dec	inc	dec

So, $f(x)$ is increasing on $(0, e^{1/2})$ and decreasing on $(e^{1/2}, \infty)$

(b) Identify local extrema

$\left(e^{+1/2}, f(e^{1/2})\right) = \left(e^{1/2}, \frac{\log(e^{1/2})}{(e^{1/2})^2}\right) = \left(e^{1/2}, \frac{1}{2e}\right)$ is a local max.

Whether it is a global max will be determined later.

(c) Find intervals of concavity

$$f''(x) = \frac{x^3\left(\frac{-2}{x}\right) - (1-2\log(x))3x^2}{x^6} = \frac{-2x^2 - 3x^2 + 6x^2\log(x)}{x^6} = \frac{-5+6\log(x)}{x^4}$$

$f''(x)$ DNE when $x=0$ $f''(x)=0$ when $-5+6\log(x)=0 \Rightarrow x=e^{5/6}$

	$(0, e^{5/6})$	$(e^{5/6}, \infty)$
$f''(x)$	−	+
ccu/D	CCD	ccu

So, $f(x)$ is concave down on $(0, e^{5/6})$ and concave up on $(e^{5/6}, \infty)$

(d) Find inflection points

$\left(e^{5/6}, \frac{5/6}{(e^{5/6})^2}\right) = \left(e^{5/6}, \frac{5}{6e^{5/3}}\right)$ is an inflection point of $f(x)$.

(e) Find horizontal asymptotes

$\lim_{x\to\infty} \frac{\log(x)}{x^2} = \lim_{x\to\infty} \frac{1/x}{2x} = \lim_{x\to\infty} \frac{1}{2x^2} = 0$. ∴ So, V.A at $y=0$.

• l'hôpital's ok since $\lim_{x\to\infty} \log(x) = \infty = \lim_{x\to\infty} x^2$

Don't need to check $x \to -\infty$ since domain is $(0,\infty)$.

(f) Find vertical asymptotoes

Only possible vertical asymptote at $x=0$ since $f(0)$ not defined.
Only need $\lim_{x\to0^+} f(x)$ since $f(x)$ not defined for $x<0$.

$\lim_{x\to0^+} \frac{\log(x)}{x^2} = -\infty$. So, vertical asymptote at $x=0$.

(g) Sketch the graph