

Vantage MATH 100 Jeopardy!

Theorems	Series and Sequences	Derivatives	Applications
<u>\$100</u>	<u>\$100</u>	<u>\$100</u>	<u>\$100</u>
<u>\$200</u>	<u>\$200</u>	<u>\$200</u>	<u>\$200</u>
<u>\$300</u>	<u>\$300</u>	<u>\$300</u>	<u>\$300</u>
<u>\$400</u>	<u>\$400</u>	<u>\$400</u>	<u>\$400</u>

Final Jeopardy

Theorems - \$100

- The Extreme Value Theorem guarantees the existence of _____, assuming that the function being examined is _____ on a _____.
- What are **global extrema**? The function must be **continuous** on a **closed interval**.



Theorems - \$200

- This theorem guarantees the existence of output values for a function, assuming the function is continuous.
- What is the Intermediate Value Theorem?



Theorems - \$300

- Assuming the function is continuous on $[a,b]$, the Mean Value Theorem guarantees the existence of this.
- What is NOTHING? :P The Mean Value Theorem requires the function be differentiable! Without differentiability, the MVT is meaningless!



Theorems - \$400

- This is the most fundamental (basic) definition of a series converging.
- What is a series converges if the sequence of partial sums converges?



Series and Sequences - \$100

- This series converges to $\frac{2}{3}$
- In general, we could take $\sum_{n=1}^{\infty} ar^{n-1}$ where $\frac{2}{3} = \frac{a}{1-r}$ and $|r| < 1$. An explicit example would be $\sum_{n=1}^{\infty} \frac{1}{3} \left(\frac{1}{2}\right)^{n-1}$



Series and Sequences - \$200

- True or False: If a sequence is bounded and we extend its domain to be positive real numbers, this function is also bounded.
- False! Take the sequence $a_n = \frac{1}{n}$. This sequence is bounded, but the function $f(x) = \frac{1}{x}$ is not bounded.



Series and Sequences - \$300

- If the first 10000 terms of a series are really big, then the series must diverge.
- False! Take a series where the first 10000 terms are 100. Then, take the remainder of the terms to be of the form $\left(\frac{1}{2}\right)^n$. The latter definitely converges, and the first 10000 terms add up to something *finite*. Hence, the whole series must converge



Series and Sequences - \$400

- This is the formal definition of a sequence a_n converging to L
- What is: a_n converges to L if for every $\varepsilon > 0$ there exists a number N so that for every $n > N$, $|a_n - L| < \varepsilon$?



Derivatives - \$100

- The chain rule is used to take the derivative of this type of function

- What is a **composition of functions**?



Derivatives - \$200

- In order to take the derivative of $\frac{\sin(\cos(x^2))}{x^3+x+8}$, we must use these three rules.
- What are the quotient rule, the power rule, and the chain rule?



Derivatives - \$300

- True or False: $\frac{d}{dx} \log \pi = \frac{1}{\pi}$. Provide justification.
- False because $\log \pi$ is a constant, so the derivative with respect to x should be 0.



Derivatives - \$400

- True or False: L'Hospital's Rule can be used on any limit of the form $f(x)/g(x)$. Give justification as to why.
- False. L'Hospital's rule requires that the limits of both $f(x)$ and $g(x)$ be zero or infinity. Using L'Hospital's rule on $\lim_{x \rightarrow 2} \frac{2x}{x^2+5}$ would yield an incorrect limit.



Applications - \$100

- This type of problem is an application of the chain rule and implicit differentiation.

- What are **Related Rates**?



Applications - \$200

- A student solving an optimization problem says that the maximum value occurs at $x=5$ since the function's value there is larger than function's value at the endpoints. They are applying this **theorem** and assuming the function is **this**.
- What is the **Extreme Value Theorem** and **continuous**?



Applications - \$300

- The comparison test for the convergence of series can only be used if the terms of both series are positive. These two series justify why it fails if the terms are not.
- Consider the negative harmonic series, $-1/n$. Note that $-1/n < 1/n!$, but although the series of $1/n!$ converges, we know that $-1/n$ does not converge.



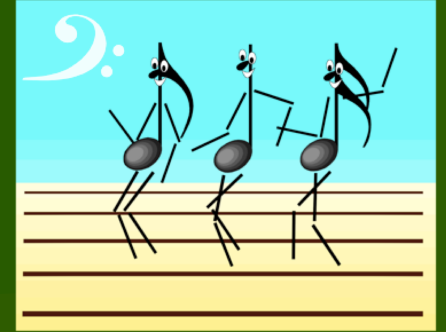
Applications - \$400

- These types of application problems are a direct consequence of curve sketching

- What are **optimization** problems?



Final Jeopardy



- These are the names of all six of the instructors in MATH 100.
- Who are Dr. Leung, Vanessa, Megan, Pam, Chan, and Jasmine! We congratulate you on finishing your first semester at UBC!

