SOLUTION SKETCHES.

Integration Techniques, Week 2, January 26, 2018

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1. Evaluate the following integrals:

(a) $\int_1^e \cos(\log(x))\,dx$ Set $u = \log(x)$, $x = e^u$ $dx = e^u du$

$= \int_0^1 \cos(u)\, e^u\, du$. Use integration by parts twice to solve for $\int_0^1 \cos(u) e^u du$

(b) $\int_1^2 \frac{4x^2+2x-1}{x^3+x^2}\,dx = \int_1^2 \frac{4x^2+2x-1}{x^2(x+1)}\,dx = \int_1^2 \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}\,dx$

After solving, get $A=3, B=-1, C=1$. $= \int_1^2 \frac{1}{x} + \frac{-1}{x^2} + \frac{1}{x+1}\,dx$

$= \log|x|\Big|_1^2 + \left(\frac{1}{x}\right)\Big|_1^2 + \log|x+1|\Big|_1^2$

(c) $\int_0^1 \frac{1}{4+4x^2+x^4}\,dx = \int_0^1 \frac{1}{(2+x^2)^2}\,dx$ You could use partial fractions, but it isn't easy because it is a repeated quadratic in the denominator.

Set $x = \sqrt{2}\tan\theta$
$dx = \sqrt{2}\sec^2\theta\, \cancel{d\theta}$ $= \int_0^{\pi/4} \frac{\sqrt{2}\sec^2\theta\, d\theta}{(2+2\tan^2\theta)^2}$

$= \int_0^{\pi/4} \frac{\sqrt{2}}{4}\frac{\sec^2\theta\, d\theta}{\sec^4\theta\, d\theta} = \frac{\sqrt{2}}{4}\int_0^{\pi/4} \cos^2\theta\, d\theta = \frac{\sqrt{2}}{8}\int_0^{\pi/4} 1+\cos(2\theta)\,d\theta$

(d) $\int_0^\infty \frac{1}{e^x+e^{-x}}\,dx$

$= \int_0^\infty \frac{e^x}{e^{2x}+1}\,dx = \lim_{b\to\infty} \int_0^b \frac{e^x}{e^{2x}+1}\,dx$ Set $u = e^x dx$ $x: 0 \to b$
$du = e^x\,dx$ $u: 0 \to e^b$

$= \lim_{b\to\infty} \int_0^{e^b} \frac{1}{u^2+1}\,du = \lim_{b\to\infty} \arctan(u)\Big|_0^{e^b}$

$= \lim_{b\to\infty} \arctan(e^b) - \arctan(0) = \frac{\pi}{2}$

(e) $\int_6^7 x\sqrt{x-5}\,dx$

Set $u = x$ $du = dx$
$dv = \sqrt{x-5}$ $v = \frac{2}{3}(x-5)^{3/2}$

$\int_6^7 x\sqrt{x-5}\,dx = x\cdot\frac{2}{3}(x-5)\Big|_6^7 - \int_6^7 \frac{2}{3}(x-5)^{3/2}dx$

(f) $\int_0^1 \frac{x^3}{\sqrt{4+x^2}}\,dx = \int_0^1 \frac{x^2}{\sqrt{4+x^2}}\cdot x\,dx$

Set $u = 4+x^2$ $du = 2x\,dx$
$x^2 = u-4$

$= \int_4^5 \frac{u-4}{\sqrt{u}}\,\frac{du}{2} = \frac{1}{2}\int_4^5 u^{1/2} - 4u^{-1/2}\,du.$