QUIZ 1

First name: SOLUTION

Last name:

Student number:

Recitation section:

Consider the integral

$$\int_0^2 (x^2 - 1)dx.$$

1. Find the Riemann sum for this integral using **right** endpoints and 4 rectangles.

$$\Delta x = \frac{2-0}{4} = \frac{1}{2}$$

$$x_i^* = 0 + i \cdot \Delta x = \frac{i}{2}, \text{ for } i = 1, 2, 3, 4$$

$$\text{So, } \sum_{i=1}^{4} f(x_i^*)\Delta x = \sum_{i=1}^{4}\left[\left(\frac{i}{2}\right)^2 - 1\right]\cdot\frac{1}{2} = \frac{1}{2}\left[0 - \frac{3}{4} + \frac{5}{4} + 3\right] = \frac{7}{4}$$

2. Given that $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$, find the Riemann sum for this integral using **right** endpoints and $n$ rectangles.

$$\Delta x = \frac{2}{n}, \quad x_i^* = \frac{2i}{n}$$

$$RS = \sum_{i=1}^{n}\left[\left(\frac{2i}{n}\right)^2 - 1\right]\cdot\frac{2}{n} = \frac{2}{n}\sum_{i=1}^{n}\left[\frac{4i^2}{n^2} - 1\right]$$

$$= \frac{8}{n^3}\left(\sum_{i=1}^{n} i^2\right) - \frac{2}{n}\left(\sum_{i=1}^{n} 1\right) = \frac{8}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right) - \frac{2}{n}\cdot n$$

$$= \frac{8(n+1)(2n+1)}{6n^2} - 2$$

3. Evaluate the limit as $n \to \infty$ of the Riemann sum in (2).

$$\lim_{n\to\infty} \frac{8(n+1)(2n+1)}{6n^2} - 2 = \frac{8}{3} - 2 = \frac{2}{3}$$