

QUIZ 2

First name: SOLUTION

Last name:

Student number:

Recitation section:

If $F(x) = \int_1^x f(t) dt$, where

$$f(t) = \int_1^{t^3} \frac{\sqrt{8+u^4}}{u} du,$$

find $F''(1)$.

$$F(x) = \int_1^x f(t) dt = \int_1^x \left[\int_1^{t^3} \frac{\sqrt{8+u^4}}{u} du \right] dt$$

$$\text{By the FTC, } F'(x) = f(x) = \int_1^{x^3} \frac{\sqrt{8+u^4}}{u} du$$

Using the FTC again, we have:

$$\begin{aligned} F''(x) = f'(x) &= \frac{d}{dx} \left[\int_1^{x^3} \frac{\sqrt{8+u^4}}{u} du \right] = f(x^3) \cdot 3x^2 \\ &= \frac{\sqrt{8+x^7}}{x^3} \cdot 3x^2 \\ &= \frac{3\sqrt{8+x^7}}{x} \end{aligned}$$

$$\text{Finally, we have that } F''(1) = \frac{3\sqrt{8+1^7}}{1} = 9$$