QUIZ 4

First name: $\mathcal{SOLUTIONS}$

Last name:

Student number:

Recitation section:

1. Give the correct form of the partial fraction decomposition for

$$\frac{x^2 + 9}{(x - 4)^3 \underbrace{(x^2 - 9)}_{(x+3)(x-3)} \underbrace{(x^2 + x + 4)}}.$$

Discriminant $= 1^2 - 4(1)(4) < 0$
$\Rightarrow$ irreducible

(Do not determine the numerical values of the coefficients.)

$$\frac{A}{x-4} + \frac{B}{(x-4)^2} + \frac{C}{(x-4)^3} + \frac{D}{x+3} + \frac{E}{x-3} + \frac{Fx+G}{x^2+x+4}$$

2. Find the antiderivative (indefinite integral)

$$\int \frac{3x^2 - 5x + 1}{(x - 3)(x^2 + 4)}\, dx.$$

$$\int \frac{3x^2 - 5x + 1}{(x-3)(x^2+4)}\, dx = \int \frac{A}{x-3} + \frac{Bx+C}{x^2+4}\, dx$$

$A(x^2+4) + (Bx+C)(x-3) = 3x^2 - 5x + 1$
• if $x=3$, $A(9+4) = 13$, so $\boxed{A=1}$
So, $(x^2+4) + (Bx+C)(x-3) = 3x^2 - 5x + 1$
$(B+1)x^2 + (-3B+C)x - 3C + 4 = 3x^2 - 5x + 1$
Hence, $B+1 = 3$ and $-3C+4 = 1$
$\boxed{B=2}$ $\boxed{C=1}$

$$= \int \frac{1}{x-3} + \frac{2x+1}{x^2+4}\, dx$$

Let $u = x^2+4$ $\Rightarrow \int \frac{1}{u}\, du = \log|u| + c$
$du = 2x\, dx$

$$= \log|x-3| + \int \frac{2x}{x^2+4}\, dx + \int \frac{1}{x^2+4}\, dx$$

Let $u = \frac{x}{2}$ $\Rightarrow \int \frac{2}{1+u^2}\, du = 2\arctan(u) + c$
$2\, du = dx$

$$= \log|x-3| + \log|x^2+4| + \frac{1}{4}\int \frac{1}{\left(\frac{x}{2}\right)^2+1}\, dx$$

$$= \log|x-3| + \log|x^2+4| + \frac{1}{2}\arctan\left(\frac{x}{2}\right) + C$$