QUIZ 5

First name: SOLUTION

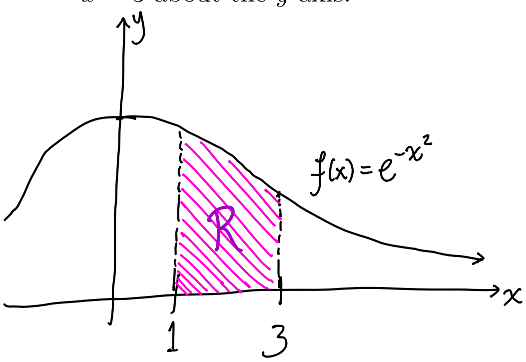
Last name:

Student number:

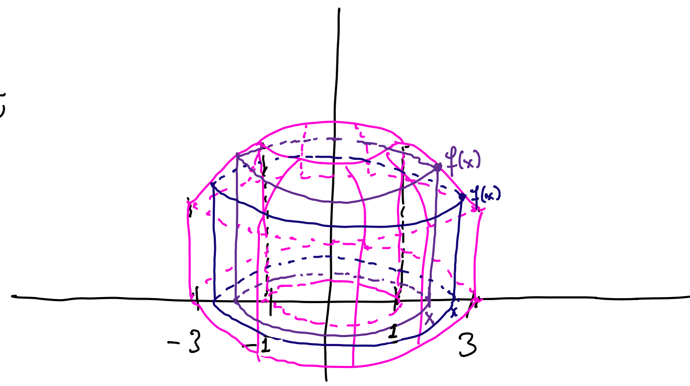
Recitation section:

---

1. Find the volume of the solid generated by rotating the region bounded by $y = e^{-x^2}$, $y = 0$, $x = 1$ and $x = 3$ about the y-axis.



Rotate about $\longrightarrow$ y-axis



SHELLS: $V = \int_1^3 2\pi r h \, dx = \int_1^3 2\pi x f(x) \, dx$

$= \int_1^3 2\pi x e^{-x^2} dx$

Let $u = x^2$

$du = 2x\, dx$

$x: 1 \to 3$

$u: 1 \to 9$

$= \int_1^9 \pi e^{-u} du$

$= -\pi e^{-u}\Big|_1^9 = -\pi e^{-9} + \pi e^{-1}$

NOTE: A lot of students wrote the following:

$\underbrace{\int_a^b -2xe^{-x^2} dx = \int_a^b e^u du}$ $= e^u\Big|_a^b = \underbrace{e^{-x^2}\Big|_a^b}_{*}$

- This is not correct.
- Mostly everyone changed to $*$, which is good, but the limits in the first integral are in terms of x. So, once we make the substitution $u = -x^2$, the limits should change too. That is, if $x: a \to b$, then $u: -a^2 \to -b^2$. So, $\int_a^b -2xe^{-x^2} dx = \int_{-a^2}^{-b^2} e^u du$

QUIZ 5

First name:

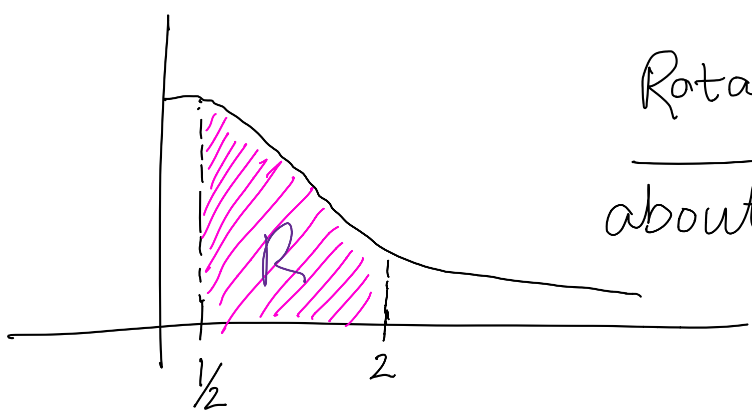
Last name:

Student number:

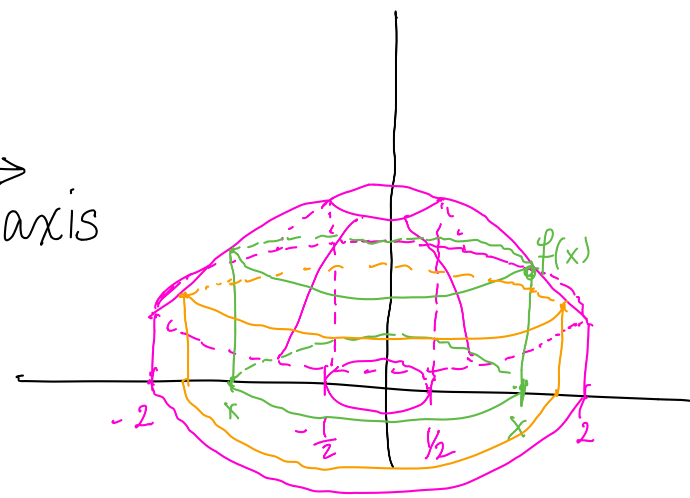
Recitation section:

---

1. Find the volume of the solid generated by rotating the region bounded by $y = e^{-x^2}$, $y = 0$, $x = \frac{1}{2}$ and $x = 2$ about the y-axis.



Rotate
$\longrightarrow$
about y-axis



SHELLS: $V = \int_{1/2}^{2} 2\pi r \cdot h\, dx = \int_{1/2}^{2} 2\pi x f(x)\, dx$

$$= \int_{1/2}^{2} 2\pi x e^{-x^2} dx$$

Let $u = x^2$
$du = 2x\, dx$
$x: 1/2 \to 2$
$u: 1/4 \to 4$

$$= \int_{1/4}^{4} \pi e^{-u} du$$

$$= -\pi e^{-u}\Big|_{1/4}^{4} = -\pi e^{-4} + \pi e^{-1/4}$$

★ SEE SAME REMARK AS ABOVE.

- $\int_{1/2}^{2} -2xe^{-x^2} dx = \int_{1/4}^{4} e^{u} du \neq \int_{1/2}^{2} e^{u} du$
- if you make a substitution, make sure to change your limits too.