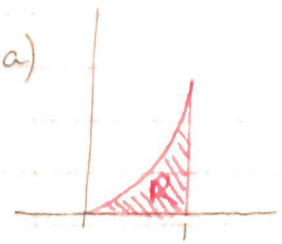
Solutions to Volumes Worksheet

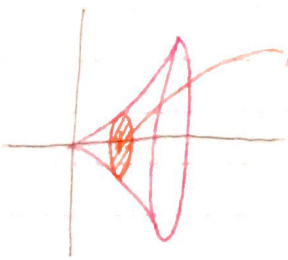
1a) $V = \int_a^b A(x)\,dx$, where $A(x)$ is the area of the slice at x.

b) $V = \int_a^b 2\pi r h\,dx$, where r is the radius of the shell and h is the height

2a)



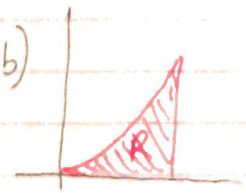
rotate about → x-axis



$A(x) = \pi r^2 = \pi(x^2)^2$

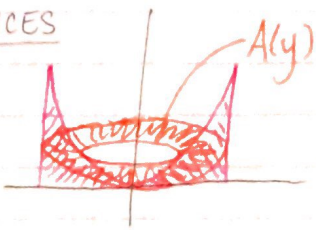
$V = \int_0^1 \pi x^4\,dx = \frac{\pi}{5}$

2b)



rotate about → y-axis

SLICES

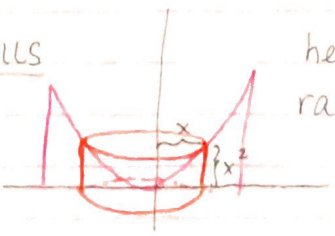


$A(y) = \pi(1^2 - (\sqrt{y})^2)$

$V = \int_0^1 \pi(1-y)\,dy$

$= \pi/2$

SHELLS

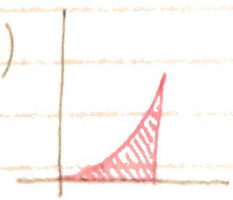


height = x^2

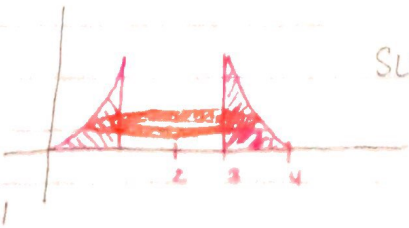
radius = x

$V = \int_0^1 2\pi x \cdot x^2\,dx = \frac{\pi}{2}$

2c)



rotate about → x = 2



SLICES: $A(y) = \pi((2-\sqrt{y})^2 - 1)$

$V = \int_0^1 \pi((2-\sqrt{y})^2 - 1)\,dy$

$= \frac{5\pi}{6}$

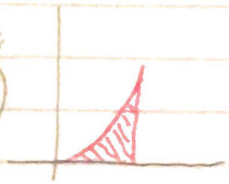
SHELLS: height: x^2

radius: $2-x$

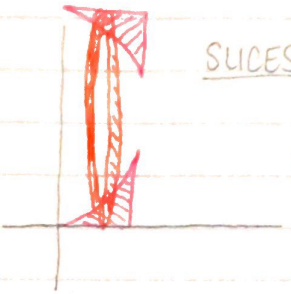
$V = \int_0^1 \pi 2(2-x)x^2\,dx = \frac{5\pi}{6}$

SOLUTIONS

2d)



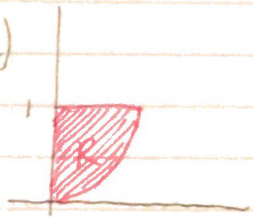
rotate → about $y=2$



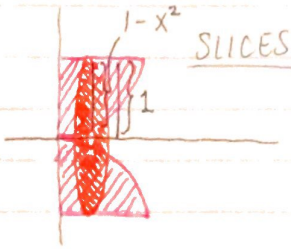
SLICES $A(x) = \pi\left(2^2 - (2-x^2)^2\right)$

$$V = \int_0^1 \pi\left(4-(2-x^2)^2\right) = \frac{17\pi}{15}$$

3a)



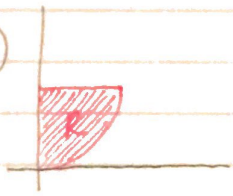
rotate → about x-axis



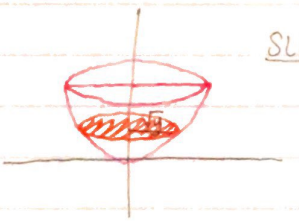
SLICES: $A(x) = \pi\left(1^2 - (x^2)^2\right)$

$$V = \int_0^1 \pi(1-x^4)\,dx = \frac{4\pi}{5}$$

3b)



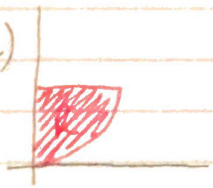
rotate about → y-axis



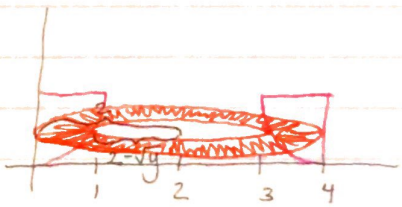
SLICES: $A(y) = \pi(\sqrt{y})^2$

$$V = \int_0^1 \pi y\,dy = \frac{\pi}{2}$$

3c)

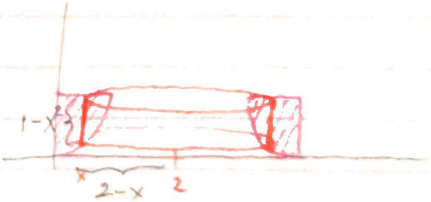


rotate about → $x=2$



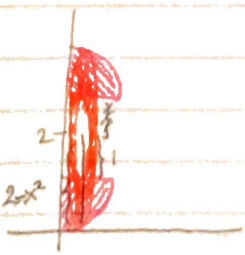
SLICES $V = \int_0^1 \pi\left(2^2 - (2-\sqrt{y})^2\right) dy = \frac{13\pi}{6}$

SHELLS: $V = \int_0^1 2\pi(2-x)(1-x^2)\,dx = \frac{13\pi}{6}$



3d)

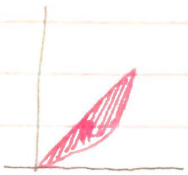
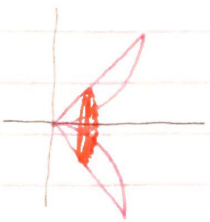
rotate → about $y=2$



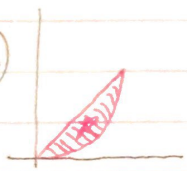
SLICES

$$V = \int_0^1 \pi\left((2-x^2)^2 - 1\right) dx = \frac{23\pi}{15}$$

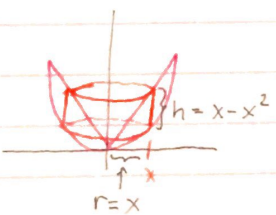
SOLUTIONS

4a)  rotate about → x-axis 

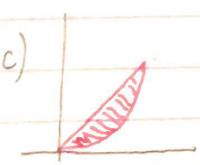
SLICES: $V = \int_0^1 \pi\left(x^2 - (x^2)^2\right) dx = \frac{2\pi}{15}$

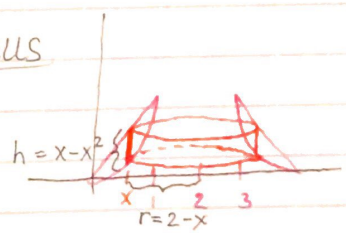
4b)  rotate about → y-axis 

SLICES $V = \int_0^1 \pi\left((\sqrt{y})^2 - y^2\right) dy = \pi/6$

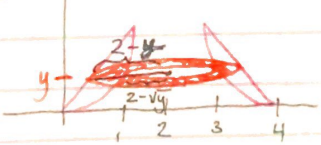


SHELLS $V = \int_0^1 \pi x(x - x^2) dx = \pi/6$

4c)  rotate about → x = 2

SHELLS 

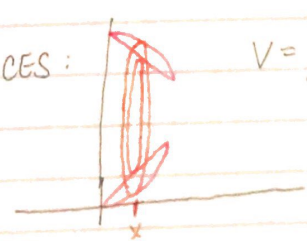
$V = \int_0^1 2\pi(2 - x)(x - x^2) dx = \frac{\pi}{2}$

SLICES 

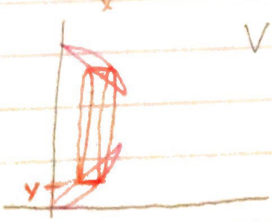
$V = \int_0^1 \pi\left((2 - y)^2 - (2 - \sqrt{y})^2\right) dy$

$= \pi/2$

4d)  rotate about → y = 2

SLICES: 

$V = \int_0^1 \pi\left((2 - x^2)^2 - (2 - x)^2\right) dx = \frac{8\pi}{15}$



SHELLS

$V = \int_0^1 2\pi(2 - y)(\sqrt{y} - y) dy = \frac{8\pi}{15}$