QUIZ 8

First name: SOLUTIONS

Last name:

Student number:

Recitation section:

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1. By recognizing the series as a Taylor (or Maclaurin) series evaluated at a particular value of x, find the sum of $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{(2n+1)!\, 2^{2n+1}} = \frac{\pi}{2} - \frac{\pi^3}{3!\, 2^3} + \frac{\pi^5}{5!\, 2^5} - \frac{\pi^7}{7!\, 2^7} + \ldots$.

Note that $\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$

So, $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{(2n+1)!\, 2^{2n+1}} = \sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{\pi}{2}\right)^{2n+1}}{(2n+1)!} = \sin\left(\frac{\pi}{2}\right) = 1$

2. Find the first three terms in the alternating series representation of the integral $\int_0^1 \frac{18}{x^2+9}\, dx$, obtained by using first three nonvanishing terms in the Maclaurin series of the integrand $f(x) = \frac{18}{x^2+9}$.

$$\int_0^1 \frac{18}{x^2+9}\, dx = \frac{18}{9}\int_0^1 \frac{1}{1-\left(\frac{-x^2}{9}\right)} dx = 2\int_0^1 \sum_{n=0}^{\infty} \left(\frac{-x^2}{9}\right)^n dx$$

$$= 2\int_0^1 \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{9^n}\, dx$$

$$= 2\left[\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{9^n(2n+1)}\right]_0^1 = 2\sum_{n=0}^{\infty} \frac{(-1)^n}{9^n(2n+1)}$$

$$= 2\left[1 - \frac{1}{9\cdot 3} + \frac{1}{81\cdot 5} - \ldots\right]$$

First three non-zero terms are $2, \frac{-2}{27}, \frac{2}{405}$

QUIZ 8

First name: Common Mistakes

Last name:

Student number:

Recitation section:

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1. By recognizing the series as a Taylor (or Maclaurin) series evaluated at a particular value of x, find the sum of $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{(2n)!\, 2^{2n}} = 1 - \frac{\pi^2}{2!\, 2^2} + \frac{\pi^4}{4!\, 2^4} - \frac{\pi^6}{6!\, 2^6} + \dots$.

The most common mistake in this problem was forgetting about the 2^{2n} in the denominator and not combining the common exponent for x and 2.

2. Find the first three terms in the alternating series representation of the integral $\int_0^1 \frac{27}{x^2+9}\, dx$, obtained by using first three nonvanishing terms in the Maclaurin series of the integrand $f(x) = \frac{27}{x^2+9}$.

This problem could be done by recognizing f as a composition of a geometric series or by recognizing the integral as having to do with arctan(x)

① Those who did arctan(x), watch out for constants in your integral

② For both strategies, remember this is a definite integral, so the terms you found shouldn't have had x's in them. You needed to evaluate your power series from 0 to 1.