

Taylor Series, March 14, 2018

1. Taylor's Theorem with Lagrange Remainder:

See class notes

2. Show that the Taylor series for $\sin(x)$ converges to $\sin(x)$.

See class notes

3. Find a power series which converges to $x \sin(x^3)$. What is its radius of convergence?

$\sin(u) = \sum_{n \geq 0} \frac{(-1)^n}{(2n+1)!} u^{2n+1}$, for all $x \in \mathbb{R}$ and substitute $u = x^3$. Radius of convergence does not change.

4. Find a power series that converges to $\arctan(5x)$. What is its radius of convergence?

Use $\arctan(u) = \sum_{n \geq 0} \frac{(-1)^n}{2n+1} u^{2n+1}$, for $|u| < 1$ and make the substitution of $u = 5x$

5. Find a series that converges to $x \log(2-x)$. What is its radius of convergence?

Use $\log(1+x) = \sum_{n \geq 1} \frac{(-1)^{n-1}}{n} x^n$, for $-1 < x \leq 1$ and properties of logarithms