QUIZ 9

First name: SOLUTION
Last name:
Student number:
Recitation section:

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1. Solve the differential equation $\frac{dy}{dx} = \frac{5x}{y}$.

This is a separable differential equation, so we aim to gather common variables on separate sides.

$$\frac{dy}{dx} = \frac{5x}{y}$$

$$y\,dy = 5x\,dx$$

$$\int y\,dy \quad \int 5x\,dx$$

$$\frac{y^2}{2} = \frac{5x^2}{2} + C \Rightarrow y^2 = 5x^2 + 2C$$ is the family of solutions to the differential equations.

2. Find the general solution, $y(t)$, which solves the problem below, by the method of integrating factors.

$$4t\frac{dy}{dt} + y = t^2.$$

Once we divide both sides by $4t$, we have a linear differential equation of the form $\frac{dy}{dt} + p(t)y = q(t)$, where $p(t) = \frac{1}{4t}$ and $q(t) = \frac{t}{4}$

Taking our integrating factor to be $I = e^{\int p(t)\,dt} = e^{\int \frac{1}{4t}\,dt} = e^{\frac{1}{4}\log(t)} = t^{1/4}$, we have

$$t^{1/4}\left(\frac{dy}{dt} + \frac{1}{4t}y\right) = \left(\frac{t}{4}\right)t^{1/4}$$

$$\frac{dy}{dt}t^{1/4} + \frac{1}{4}t^{-3/4}y = \frac{t^{5/4}}{4}$$

$$\int \frac{d}{dt}\left(t^{1/4}\cdot y\right)dt = \int \frac{t^{5/4}}{4}\,dt$$

$$t^{1/4}y = \frac{t^{9/4}}{9} + C \Rightarrow y = \frac{t^2}{9} + Ct^{-1/4}$$ is the family of solutions to the ODE.

QUIZ 9

First name: Observed Common Mistakes.

Last name:

Student number:

Recitation section:

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1. Solve the differential equation $\dfrac{dy}{dx} = \dfrac{4x}{y}$.

Generally, well done.

Only common mistake involved the constant. Either

① it was completely forgotten

or ② The relationship between x, y, & C was lost.

- Some students wrote:

$$\frac{1}{2}y^2 = \frac{5x^2}{2} + C \Rightarrow y^2 = 5x^2 + C$$

- This is not 100% wrong; yes, it is still a constant, but it is not the SAME C. This could be avoided by naming it C_1 or just something different.

2. Find the general solution, $y(t)$, which solves the problem below, by the method of integrating factors.

$$5t\,\frac{dy}{dt} + y = t^2.$$

A few major issues came up here, number 1 being not knowing the steps of the technique. These problems are not complicated; you are basically following a recipe. Look at your notes, memorize the steps, and do more problems.

Second, was computing the integrating factor.

- Some claimed $I = e^{\int \frac{dt}{4t}} = e^{\log|4t|}$. A bit of reflection should help you realize this is wrong. If $\int \frac{1}{4t}\,dt = \log|4t|$, then $\frac{d}{dt}(\log(4t)) = \frac{1}{4t}$. But, you know that by the chain rule, $\frac{d}{dt}(\log(4t)) = \frac{1}{4t}\cdot 4 = \frac{1}{t}$

Third, if you did compute I incorrectly, it should be obvious later in your problem solving. The entire point of multiplying $y' + p(x)y = q(x)$ by I is so that $I(y' + p(x)y) = \frac{d}{dx}(Iy)$. If you compute $\frac{d}{dx}(Iy)$ for the I you found and it does not equal your original equation of ①, you 100% did something wrong.