Differential Equations II, March 23, 2018

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1. Suppose a reservoir contains 1000 L of polluted water, in which is dissolved 10 kg of pollutant. Suppose polluted water flows into the tank at a rate of 10 L/min, with 0.1 kg of pollutant per litre. The reservoir is contatnly and thoroughly mixed, and the solution flows out at a rate of 10 L/min. Find the amount of pollutant in the tank at any time t.

   SOLUTION: Let $p(t)$ be the amount of pollutant in the tank at any time t. We know that the rate of change of the amount of pollutant in the tank is equivalent to the rate in minus the rate out. So:

$$\begin{aligned} \frac{dp}{dt} &= p_{in} - p_{out} \\ &= 0.1\frac{kg}{l} \cdot 10\frac{l}{min} - \frac{p}{1000}\frac{kg}{l} \cdot 10\frac{l}{min} \\ &= 1 - \frac{p}{100} \end{aligned}$$

   This is a separable differential equation and straightforward to solve, with the intitial consition being that $p(0) = 10$.

SOLVE THE FOLLWOING DIFFERENTIAL EQUATIONS:

1. $\dfrac{dy}{dx} + 2y = x$

   See class notes

2. $\dfrac{1}{x}\dfrac{dy}{dx} - \dfrac{2}{x^2}y = x\cos(x)$

   See class notes

3. $x\dfrac{dy}{dx} - 2y = 10x^2$, with $y(1) = 3$

   SOLUTION: Putting this equation into standard form yields: $\dfrac{dy}{dx} - \dfrac{2y}{x} = 10x$. We take our integrating factor to be $I = e^{\int \frac{2}{x}\,dx} = x^2$. Multiplying both sides of the *standard form* of the original differential equation yeilds:

$$\begin{aligned} \frac{dy}{dx}x^2 - 2xy &= 10x^3 \\ \frac{d}{dx}\left(x^2y\right) &= 10x^3 \\ \int \frac{d}{dx}\left(x^2y\right)\,dx &= \int 10x^3\,dx \\ x^2y &= 10\frac{x^4}{4} + c \\ y &= \frac{5x^2}{2} + \frac{c}{x^2} \end{aligned}$$

   Since $y(1) = 3$, we have that $c = \frac{1}{2}$ and the final solution is $y = \dfrac{5x^2}{2} + \dfrac{1}{2x^2}$

4. Summarize the method of integrating factors.

   This is something for you to do on your own. Some questions that might help you are: When should you use this method? What is the purpose of it? Why do we use it? What are the steps to the method?