Differential Equations IV, April 4, 2018

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1. Find the orthogonal trajectories of the family of curves defined by $y^5 = kx^2$.
   See solution from last week.

2. Into a 2000 liter container is placed 1000 liters of a brine solution containing 40 kg of salt. A brine solution containing .02 kg/l of salt flows into the container at a rate of 50 l/min. The solution is kept thoroughly mixed, and the mixture flows out at a rate of 25 l/min. How much salt is in the container at the moment it overflows?

   Once again, we have that the the rate of change is rate in minus rate out. So, we have

   $$R_{in} = (0.2\frac{kg}{l})(50\frac{l}{min}) = 1\frac{kg}{min}$$

   Finding R_{out} is a little tricky because inflow is greater than outflow. How much solution is in the tank at any time t? After 1 minute, there are 1000 litres + 50 litres in - 25 litres out, so 1025 litres. After two minutes, we have 1000 + 100 litres in minus 50 litres out, so 1050 litres. In total, after t minutes, we have $1000 + 25t$ litres of solution in the tank. Hence,

   $$R_{out} = \frac{s}{1000 + 25t}\frac{kg}{l} \cdot \frac{25l}{min} = \frac{s}{40 + t}$$

   Finally, the differential equation we have is $\frac{ds}{dt} + \frac{s}{40+t} = 1$. This is a linear differential equation which is solvable by integrating factors. The solution, once subbing in the initial value of $s(0) = 40$, is $s(t) = \frac{80t + t^2}{2(40+t)} + \frac{40}{40+t}$.

3. Solve the differential equation $\frac{dy}{dx} - \frac{y}{3x} = e^x y^4$.

   Notice that this looks approximately like a linear differential equation, except for the y$^4 ontheLHS.Setting$u= y^{-3}, we have that $du/dx = -3y^{-4}dy/dx$. Then, after a little bit of manipulation, we have the differential equation $du/dx + u/x = -3e^x$, which can be solved using integrating factors. In closing, $y^3 = \frac{x}{e^x - xe^x + C}$.