**Effectiveness of Math Manipulatives in Classrooms: Annotated Bibliography**

**Introduction**

Learning Math is not easy, teaching Math is just as hard. Grade schools across Canada are starting to shift from traditional rote math learning environments and methods to reformed and innovative classrooms with materials that are innovative and modern, and use pedagogies that helps students make connections in Math and other subjects like literacy. With these new methods and approaches, a visible increase in the amounts of manipulatives and “tools” are becoming readily available for teachers to use for supporting lessons. These technologies can range from virtual manipulatives to concrete manipulatives. With this increase in quantity for useable materials to support teaching and lesson learning, the question of whether they are useful comes into hand. In some Canadian provinces, their math curriculum uses a series of textbooks call Math Makes Sense, that teaches big ideas and key concepts in math literacy by focusing on word problem solving. Problem solving skill development seems to play bigger roles in childhood development now as such methods are considered to nurture critical thinkers. This switch however has been considered quite different from the traditional methods of rote teaching that Math teachers were taught in and hence be more familiar with.

Education companies like the ones that published the textbooks used in schools and education researchers are also developing more methods and approaches for such innovative teaching environments. Most importantly creative new virtual manipulatives are starting to emerge to personalize math learning and support math literacy in school and at home but there are also many uncertainties about it’s effectiveness and use. The articles below were selected with the intention to present the trends and research directions for the use math manipulatives in these enhanced classrooms and some seen results from the studies conducted across the globe.

**Articles**

Bouck, E. C., & Flanagan, S. M. (2010). Virtual manipulatives: What they are and how teachers

can use them. Intervention in School and Clinic, 45(3), 186-191.

The research authors of this article looked at the use of virtual manipulatives when used in classrooms with high-incidence disabilities and regular students, and how the use of manipulatives in such an environment has shown positive results and has great potential in supporting math learning. The article first introduced the difference between math manipulatives in used now, concrete and virtual. Concrete manipulatives are physical objects that students can manipulate throughout the exploration and understanding development process, whereas virtual manipulatives are defined to be “computer-based simulations of physical manipulatives that are accessed via the Internet or computer software” (Bouck, 2010). The article states the importance of the interactive features that virtual manipulatives need to have, as it would prompt students for more immediate input and feedback. The authors also stated in their research that results from their study, shows virtual manipulatives to have positive impact and “are effective in facilitating students’ understanding of mathematical concepts”. It was also noted that virtual manipulatives can be used like traditional concrete manipulatives, as a support in a lesson or have actual lessons constructed around the virtual manipulative. Virtual manipulatives also allow students and teachers to access resources that may not have been readily available previously due to limited resources. Such manipulatives also would not require a lot of transition time when switching from one type of tools to another. A very important concept that the authors also mentioned in the article is the versatility of virtual manipulatives as it can be used to teach students in a group or individually and can allow for customization of lessons for students in the same classroom studying at different levels.

Kamii, C., Lewis, B. A., & Kirkland, L. (2001). Manipulatives: When are they useful? Journal of

Mathematical Behavior, 20(1), 21-31

The researchers in this article examined the usefulness of manipulatives, virtual and concrete, in the acquisition of “logicomathematical knowledge”. It suggests that such knowledge in children can be constructed through own thinking, and manipulatives encourages thinking and problem solving. Like Larkin’s article, Kamii also suggests that such uses of manipulatives are only useful during certain stages of a child’s development. Virtual manipulatives counter the practices of the traditional rote memory methods and are greatly recommended for use by the authors. This practice promotes thinking in problem solving and helps the development of logicomathematical knowledge which is the “mental relationship” between math and the physical world, in the minds of the children. The authors based their research on the acquisition of logicomathematical knowledge from Piaget’s theory of constructive abstraction. The article also presented manipulatives that can be used to promote the development of different types of thinking and mental relationships. Authors also emphasized that “manipulatives are useful or useless depending on the quality of thinking they stimulate” and mathematical abilities developed in children are not in the use of the manipulatives but in the mental connections that were developed after the use of them.

Lamberty, K. K., & Kolodner, J. L. (2002). Exploring digital quilt design using manipulatives as

a math learning tool. In Proceedings of ICLS (Vol. 2, pp. 1-7).

Lamberty wrote in this article of the use of manipulatives to help children understand tough math concepts. Learning fractions was used as an example in this article to demonstrate how students used manipulatives to help them comprehend fraction concepts. Physical representations of objects “help learners visualize math concepts in a concrete way” (Lamberty, 2002). The authors noticed that the use of manipulatives doesn’t always enhance learning and can be lacking at times as the uses don’t always engage the students in productive play needed for curriculum learning. To tackle this noticed problem, the authors developed the DigiQuilt project with hopes to find a possible solution through the design approach. They proposed that through quilt-work students can gain math understanding for topics like fractions, area and perimeter and art concepts like symmetry and balance. Students would need to design patterns using the DigiQuilt software as part of their project/assignment. The digital platform has functions like physical manipulatives and would allow learners to move shapes to create patterns on grids. The proposed math learning method also promotes artistic and personal expression, as art components are involved. This learning by design method suggested by the authors, requires frequent reflection, presentations and discussions so that students can be scaffolded correctly. The article also included details of how they completed this study and the results when conducted on grade 3 students who had about an hour of computer time every six days. As a result, from their study, the authors sided with physical manipulatives as better tools for introductory learning of math concepts though the concept of merging math and art through design projects is still consider plausible but needs work.

Larkin, K. (2016). Australian primary mathematics classroom: Mathematics education and

manipulatives: Which, when, how? Australian Association of Mathematics Teachers.

Larkin suggests in his article that a framework for the selection and use of math materials, to scaffold student at different experiential stages of learning effectively. This means that when correctly selected tools are used, students can be scaffolded through Enactive experience, Iconic Representation of experience and Symbolic representation. To achieve this, Larkin explains in the framework in his article of which materials should be used, when and why they should be used to see the best results. He noted from his research that materials were most effective when used between the ages of 7 -11 and would the manipulatives would start losing effectiveness as users age. Objects that users can relate to from the real world also showed more positive results and students who were scaffolded during use made stronger connections between the material and mathematics. Larkin’s suggested framework is designed for primary school teachers to help them decide the use of mathematic manipulatives, and help them as a guide, to facilitate student learning.

Moyer, P. S., Bolyard, J. J., & Spikell, M. A. (2002). What are virtual manipulatives? Teaching

Children Mathematics, 8(6), 372-377.

Due to the increasing use of innovative technologies to enhance teaching and learning math, the authors of this article presented a simple introduction to virtual manipulatives, what they are and how they are and could be used in classrooms. Virtual manipulatives are first defined in the article as static and dynamic visual representations of the traditionally used concrete manipulatives. Static visuals like pictures in books or on the blackboard are classified in this section while visual objects on the computer though like the static ones are considered dynamic and can be manipulated in similar ways to the concrete counterparts except it would be on the computer while static visual can’t be manipulated the same concrete way. Thus, static virtual manipulatives aren’t completely true virtual manipulatives. Virtual manipulatives defined in the article, allows users to make meaning and see relationships through their own actions/manipulations on the computer. The article emphasized greatly the importance of virtual manipulatives to be interactive and to have interactive features that can prompt users and give immediate feedback as those features are crucial in learning. Because of this importance, static virtual manipulatives like a chart on the wall, are significantly less effective as a support for enhance teaching. The authors concluded that the world of virtual manipulatives have great potential and may become the “most appropriate mathematics tool for the next generation” though work still needs to be done in this area of study and innovative approach.

Namukasa, I. K., Namukasa, I. K., & Gadanidis, G. (2011). International journal of information

and education technology: The I teach mathematics online project: Learning and teaching through innovative practices IACSIT.

The authors of this article recognized the increasing need for providing training and support to teachers learning to use web-based resources and digital technologies and reports on the development of their prototype of the I Teach Math project. A project “developed to deliver pedagogical content knowledge for teaching through problem solving” (Namukasa, 2011) The project serves as an online community with a database of resources that teachers can use as an aid. Teachers teaching math from different schools and teaching environments can share resources and tips for math teaching though videos of recorded lessons and discussions. The initiative for this prototype was due to the implementation of problem-solving skills development in curriculums and the noticeable shift to teach through problem-solving. The project was an attempt to find a solution to better train teachers for problem solving teaching lessons by connecting experienced teachers with inexperienced ones virtually.

**Conclusion**

From the articles above, it can be confirmed that math manipulatives have their benefits in supporting learning. However, there are a few factors that needs to be considered when picking the right manipulative to use for math lessons. Math manipulatives are basically categorized into concrete and virtual types. Concrete manipulatives are traditional tools used in classrooms that are easy and more familiar to use than virtual ones for many teachers but with technology advances, these concrete manipulatives, though not considered obsolete yet, are considered to be less effective when compared to their virtual counterparts. Virtual manipulatives are the new digitalize concrete manipulatives that are gaining great popularity and showing better results in enhanced learning classrooms. The effectiveness of the virtual manipulatives to help students understand and learn mathematical skills relies the interactive programming behind each tool to prompt user and provide immediate feedback. With the increasing interest, there is also a visibly increasing amounts of virtual manipulatives available online, which also allows for limitless opportunities to enhance individual learning and group learning. These new tools also broaden the limits of previously manipulative-lacking classrooms and allows for more personalized learning to occur in multi-leveled classrooms. However, it has also been pointed out in many of the articles that such effectiveness is also greatly dependent on teachers being able to select the right manipulative at the right time for the right student for the right lesson. This specific requirement might be harder for educators to achieve than it sounds. That said, the common consensus from the articles is that virtual manipulatives have great potential and is most likely the method for the new generations to come.